Sample midterm

While solving the problems of the midterm empty A4 sized papers, writing instruments can be used. No other exam aids (e. g. calculators, notes) are permitted! Also any communcation with other students will be forbidden.

There will be 6 problems for 20 points in the midterm, to pass you need to get at least 10 points. You will have 60 minutes to work.

You find below the problems from a midterm few years ago. I plan keep the same difficulty.

1. Decompose the polynomial $f(x) = x^4 + x^3 - 2x^2 + 2$ to irreducibles over $\mathbb{Q}$ and $\mathbb{Z}_3$! (4 points)

2. What is the product and the sum of the squares of the complex roots the polynomial $p(x) = x^4 - 5x^3 + 15x^2 - 20$? What is the number of the real roots? (3 points)

3. Show that $\Phi_{p^2}(x) = \Phi(x^p)$ if $p \in \mathbb{N}$ is a prime. Compute $\Phi_{27}(x)$! (3 points)

4. Depending on the parameter a how many solutions does the following system of linear equations have over $\mathbb{R}$ and $\mathbb{Z}_5$ (4 points)?

$$\begin{cases} x+ & 2y+ & 2z = & 1 \\ 6x- & 3y+ & 2z = & 1 \\ 2x+ & 4y+ & az = & a \end{cases}$$

5. For which values of p and q do the planes $x - y + pz = q$, $x + 2y = 6$ and $x - 2y + z = -3$ intersect in a line? (3 points)

6. Let $\varphi : \mathbb{R}^2 \to \mathbb{R}^2$ which first projects the plane perpendicularly to the x axis and then rotates by $45°$. What is its matrix and kernel? Compute the dimension of its image! (3 points)