Trigonometria - gyakorló feladatok

Az alábbi feladatokhoz használjuk fel a következő azonosságokat:

1) $\sin^2 x + \cos^2 x = 1$

2) $\cos 2x = \cos^2 x - \sin^2 x$

3) $\sin 2x = 2 \sin x \cos x$

1. Oldja meg az alábbi egyenleteket a valós számok halmazán:

a) $4 \sin^2 x - 8 \cos x = -1$ **b)** $\cos 2x - 2 \sin x = -3$

c) $2 \cos^2 x + \sin x - 1 = 0$ **d)** $\cos 2x - \sin^2 x - 6 \sin x = 4$

2. Oldja meg az alábbi egyenleteket a $[0,\ 2\pi]$ intervallumon:

a) $\sin 2x - \sin x = \operatorname{tg} x$ **b)** $(\cos x + \sin x)^2 + \cos x = 1$

c) $2 \sin^2 x + \cos(\pi - x) = 2$ **d)** $\cos 2x = \sin x + 1$

e) $\cos 2x + 2 \cos^2 x - 2 \sin x - 1 = 0$ **f)** $2 \sin x \cos 2x = \sin 2x$

Eredmények

1. a) $x_{1,2} = \pm \dfrac{\pi}{3} + k \cdot 2\pi \quad (k \in \mathbb{Z})$

b) $x = \dfrac{\pi}{2} + k \cdot 2\pi \quad (k \in \mathbb{Z})$

c) $x_1 = \dfrac{\pi}{2} + k \cdot 2\pi, \quad x_2 = \dfrac{7\pi}{6} + l \cdot 2\pi, \quad x_3 = \dfrac{11\pi}{6} + m \cdot 2\pi \quad (k,\ l,\ m \in \mathbb{Z})$

d) $x = -\dfrac{\pi}{2} + k \cdot 2\pi \quad (k \in \mathbb{Z})$

2. a) $x_1 = 0, \quad x_2 = \dfrac{2\pi}{3}, \quad x_3 = \pi, \quad x_4 = \dfrac{4\pi}{3}, \quad x_5 = 2\pi$

b) $x_1 = \dfrac{\pi}{2}, \quad x_2 = \dfrac{7\pi}{6}, \quad x_3 = \dfrac{3\pi}{2}, \quad x_4 = \dfrac{11\pi}{6}$

c) $x_1 = \dfrac{\pi}{2}, \quad x_2 = \dfrac{2\pi}{3}, \quad x_3 = \dfrac{4\pi}{3}, \quad x_4 = \dfrac{3\pi}{2}$

d) $x_1 = 0, x_2 = \pi, \quad x_3 = \dfrac{7\pi}{6}, \quad x_4 = \dfrac{11\pi}{6}, \quad x_5 = 2\pi$

e) $x_1 = \dfrac{\pi}{6}, \quad x_2 = \dfrac{5\pi}{6}, \quad x_3 = \dfrac{3\pi}{2}$

f) $x_1 = 0, \quad x_2 = \dfrac{2\pi}{3}, \quad x_3 = \pi, \quad x_4 = \dfrac{4\pi}{3}, \quad x_5 = 2\pi$