

Kalkulus 12. gyakorlat megoldások

2021. október 26.

1.feladat

$$a, \lim_{h \rightarrow 0} \frac{\sqrt{6(4+h)+1} - \sqrt{6(4)+1}}{h} = \lim_{h \rightarrow 0} \frac{25+6h-25}{h(\sqrt{6(4+h)+1} + \sqrt{6(4)+1})} = \lim_{h \rightarrow 0} \frac{h}{h} \frac{6}{\sqrt{25+h}+5} = \frac{6}{10}$$

$$b, \lim_{h \rightarrow 0} \frac{1/\sqrt{2(1+h)+7} - 1/\sqrt{2+7}}{h} = \lim_{h \rightarrow 0} \frac{3-\sqrt{2h+9}}{3h\sqrt{2h+9}} = \lim_{h \rightarrow 0} \frac{9-(2h+9)}{3h\sqrt{2h+9}(3+\sqrt{2h+9})} = \lim_{h \rightarrow 0} \frac{h}{h} \frac{-2}{3h\sqrt{2h+9}(3+\sqrt{2h+9})} = -\frac{1}{27}$$

$$c, \lim_{h \rightarrow 0} \frac{1/(3(-1+h)+1) - 1/(-3+1)}{h} = \lim_{h \rightarrow 0} \frac{-2-(3h-2)}{h(-2)(3h-2)} = \lim_{h \rightarrow 0} \frac{h}{h} \frac{3}{-6h+4} = \frac{3}{4}$$

$$d, \lim_{h \rightarrow 0} \frac{\sqrt[3]{0+h} - \sqrt[3]{0}}{h} = \lim_{h \rightarrow 0} h^{-\frac{2}{3}} = \infty \Rightarrow \nexists f'(a)$$

$$e, \lim_{h \rightarrow 0} \frac{\sqrt[3]{0+h} \sin(\sqrt[3]{(0+h)^2}) - \sqrt[3]{0} \sin(\sqrt[3]{(0)^2})}{h} = \lim_{h \rightarrow 0} \frac{\sin(\sqrt[3]{h^2})}{\sqrt[3]{h^2}} = 1$$

2.feladat

$$a, 4 + 6x^2 + 35x^6$$

$$b, 2\pi x^{\pi-1} + \frac{5}{4}x^{-\frac{3}{4}} + \frac{4}{5}x^{-\frac{3}{5}} - \frac{5}{7}x^{-\frac{8}{7}} - \frac{4}{3}x^{-\frac{5}{3}}$$

$$c, \frac{\sin(x)}{x} + \ln(x) \cos(x)$$

$$d, (2x + \frac{x^{-1}}{2}) \tan(x) + \frac{(x^2 + x^{\frac{1}{2}})}{\cos^2(x)}$$

$$e, \frac{-\sin(x)\sqrt{x} - \frac{1}{2}x^{-\frac{1}{2}} \cos(x)}{x}$$

$$f, \frac{\frac{1}{\sqrt{1-x^2}}(x^2+1) - 2 \arcsin(x)}{(x^2+1)^2}$$

$$g, \frac{1}{1+e^{2x}} e^x$$

$$h, \cosh(\sqrt{x}) \frac{1}{2} x^{-\frac{1}{2}}$$

$$i, \frac{1}{2} (\operatorname{arcosh}(x^2))^{-\frac{1}{2}} \frac{1}{\sqrt{x^4-1}} 2x$$

3.feladat

$$a, \cos(3x)3$$

$$b, \frac{\frac{1}{x}(5\sqrt[5]{x}-x) - (\ln 2x+1)(\sqrt[5]{x^{-4}}-1)}{(5\sqrt[5]{x-x})^2}$$

$$c, \cos(x^3)3x^2$$

$$d, 5 \sin^4(2x^3) \cos(x)6x^2 = 30 \sin^2(2x) \cos(x)x^2$$

$$e, 2x\sqrt{1+2x^4} + \frac{1}{2}(1+2x^4)^{-\frac{1}{2}} 8x^3$$

$$f, \frac{-1}{\sin^2(x)}$$

$$g, \frac{(2x3^x + x^2 3^x \ln 3)(2x^2+7) - (x^2 3^x + 3) \cdot 4x}{(2x^2+7)^2}$$

$$h, 8(x^3 + 2x^2 - 6)^7(3x^2 + 4x)$$

$$i, 3(x^3 + \cos^2(x^4))^2(3x^2 + 2 \cos(x^4)(-\sin(x^4)))4x^3$$

$$j, \frac{1}{2} \left(\frac{x+1}{x^{2021} - \frac{1}{x}} \right)^{-\frac{1}{2}} \cdot \frac{(x^{2021} - \frac{1}{x} - (x+1)(2021x^{2020} + \frac{1}{x^2}))}{(x^{2021} - \frac{1}{x})^2}$$

4.feladat

a. $f'(0) = -5$, $f'(4) = 3$. Az érintő egyenlete: $y = 10 - 5x$, $y = 3x - 6$.

b, $f'(\frac{\pi}{2}) = -2$. Az érintő egyenlete: $y = \pi - 2x$.

c, $f'(\frac{\pi}{4}) = 0$. Az érintő egyenlete: $y = 1$.

d, $f'(0) = 1$. Az érintő egyenlete: $y = x$.

e, $f'(2) = -\frac{2}{3}$. Az érintő egyenlete: $y = \frac{1}{3} - \frac{2}{3}x$.