Kalkulus 12. gyakorlat megoldások

2024. október 9.

1.feladat

a, $\lim_{h\to 0} \frac{\sqrt{6(4+h)+1}-\sqrt{6(4)+1}}{h} = \lim_{h\to 0} \frac{25+6h-25}{h(\sqrt{6(4+h)+1}+\sqrt{6(4)+1})} = \lim_{h\to 0} \frac{h}{h} \frac{6}{\sqrt{25+h}+5} = \frac{6}{10}$

b, $\lim_{h\to 0} \frac{1/\sqrt{2(1+h)+7}-1/\sqrt{2+7}}{h} = \lim_{h\to 0} \frac{3-\sqrt{2h+9}}{3h\sqrt{2h+9}} = \lim_{h\to 0} \frac{9-(2h+9)}{3h\sqrt{2h+9}(3+\sqrt{2h+9})} = \lim_{h\to 0} \frac{h}{h} \frac{-2}{3h\sqrt{2h+9}(3+\sqrt{2h+9})} = -\frac{1}{27}$

c, $\lim_{h\to 0} \frac{1/(3(-1+h)+1)-1/(-3+1)}{h} = \lim_{h\to 0} \frac{-2-(3h-2)}{h(-2)(3h-2)} = \lim_{h\to 0} \frac{h}{h} \frac{3}{-6h+4} = \frac{3}{4}$

d, $\lim_{h\to 0} \frac{\sqrt[3]{0+h}-\sqrt[3]{0}}{h} = \lim_{h\to 0} h^{\frac{-2}{3}} = \infty \Rightarrow \nexists f'(a)$

e, $\lim_{h\to 0} \frac{\sqrt[3]{0+h}\sin(\sqrt[3]{(0+h)^2})-\sqrt[3]{0}\sin(\sqrt[3]{(0)^2})}{h} = \lim_{h\to 0} \frac{\sin(\sqrt[3]{h^2})}{\sqrt[3]{h^2}} = 1$

2.feladat

a, $4+6x^2+35x^6$

b, $2\pi x^{\pi-1} + \frac{5}{4}x^{\frac{-3}{4}} + \frac{4}{5}x^{\frac{-3}{5}} - \frac{5}{7}x^{\frac{-8}{7}} - \frac{4}{3}x^{\frac{-5}{3}}$

c, $\frac{\sin(x)}{x} + \ln(x)\cos(x)$

d, $(2x + \frac{x^{\frac{-1}{2}}}{2})\tan(x) + \frac{(x^2+x^{\frac{1}{2}})}{\cos^2(x)}$

e, $\frac{-\sin(x)\sqrt{x}-\frac{1}{2}x^{\frac{-1}{2}}\cos(x)}{x}$

f, $\frac{\frac{1}{\sqrt{1-x^2}}(x^2+1)-2\arcsin(x)}{(x^2+1)^2}$

g, $\frac{1}{1+e^{2x}}e^x$

h, $\cosh(\sqrt{x})\frac{1}{2}x^{\frac{-1}{2}}$

i, $\frac{1}{2}(\operatorname{arcosh}(x^2))^{\frac{-1}{2}}\frac{1}{\sqrt{x^4-1}}2x$

3.feladat

a, $\cos(3x)3$

b, $\frac{\frac{1}{x}(5\sqrt[5]{x}-x)-(\ln 2x+1)(\sqrt[5]{x^{-4}}-1)}{(5\sqrt[5]{x}-x)^2}$

c, $\cos(x^3)3x^2$

d, $5\sin^4(2x^3)\cos(x)6x^2 = 30\sin^2(2x)\cos(x)x^2$

e, $2x\sqrt{1+2x^4} + \frac{1}{2}(1+2x^4)^{\frac{-1}{2}}8x^3$

f, $\frac{(4x^3-4^{x-1}\ln 4)(e^x+8-2\ln(x))-(x^4-4^{x-1}+3)(e^x-\frac{2}{x})}{(e^x+8-2\ln(x))^2}$

g, $\frac{-1}{\sin^2(x)}$

h, $\frac{(2x3^x+x^2 3^x\ln 3)(2x^2+7)-(x^2 3^x+3)\cdot 4x}{(2x^2+7)^2}$

i, $8(x^3+2x^2-6)^7(3x^2+4x)$

j. $3(x^3 + \cos^2(x^4))^2(3x^2 + 2\cos(x^4)(-\sin(x^4))4x^3)$

k, $\frac{1}{2}\left(\frac{x+1}{x^{2021}-\frac{1}{x}}\right)^{-\frac{1}{2}} \cdot \frac{(x^{2021}-\frac{1}{x}-(x+1)(2021x^{2020}+\frac{1}{x^2}))}{(x^{2021}-\frac{1}{x})^2}$

l, $\cosh\left(\frac{x-1}{2x+1}\right) \cdot \frac{2x+1-(x-1)\cdot 2}{(2x+1)^2}$

4.feladat

a. $f'(0) = -5,\ f'(4) = 3$. Az érintő egyenlete: $y = 10 - 5x$, $y = 3x - 6$.

b, $f'(2) = -\frac{2}{3}$. Az érintő egyenlete: $y = \frac{1}{3} - \frac{2}{3}x$.

c, $f'(\frac{\pi}{2}) = -2$. Az érintő egyenlete: $y = \pi - 2x$.

d, $f'(\frac{\pi}{4}) = 0$. Az érintő egyenlete: $y = 1$.

e, $f'(0) = 1$. Az érintő egyenlete: $y = x$.