Kalkulus 5. gyakorlat megoldások

2023. szeptember 21.

1.feladat

a, $\lim_{n\to\infty} \sqrt{2n^2+5n} - \sqrt{2n^2-1} = \frac{3}{\sqrt{2}}$

b, $\lim_{n\to\infty} \sqrt{n^4+2n^2+3} - \sqrt{n^4+n} = 1$

c, $\lim_{n\to\infty} \sqrt{4n^4+n^2-2} - 2n^2 = \frac{1}{4}$

2.feladat

$a_n = n^3 - 7n^2 + 5n + 3 \geq n^3 - 7n^2 = n^2(n-7) \overset{\text{ha } n\geq 7}{\geq} (n-7)^3 \to \infty$

$b_n = \sqrt{n^5 + 2n^2} \geq \sqrt{n^5} = n^{\frac{5}{2}} \to \infty$

$c_n = \sqrt{n^5 - 2n^2} = \sqrt{n^2(n^3-2)} \geq \sqrt{n^2(n^2-2)} \overset{\text{ha } n\geq 2}{\geq} \sqrt{(n^2-2)(n^2-2)} = n^2 - 2 \to \infty$

$d_n = (\sqrt{n^4+2n^3} - \sqrt{n^4+5n^3}) \cdot 1 = \frac{(n^4+2n^3)-(n^4+5n^3)}{\sqrt{n^4+2n^3}+\sqrt{n^4+5n^3}} = \frac{n^3}{n^2}\frac{7}{\sqrt{1+2/n}+\sqrt{1+5/n}} = n\frac{7}{\sqrt{1+2/n}+\sqrt{1+5/n}} \to \infty$

3.feladat

$a_n = \frac{n!}{2^n} = \frac{n\cdot(n-1)\cdot(n-2)...2\cdot 1}{2\cdot 2\cdot 2...2\cdot 2} \geq \frac{n}{2}\cdot 1\cdot 1...1\cdot\frac{1}{2} = \frac{n}{4} \to +\infty;$

$b_n = \frac{2^n}{n}$: $\frac{2^n}{n} > n$, ha $n > 4$, ezt teljes indukcióval lehet belátni, így a speciális rendőr-elvből következik, hogy $b_n \to +\infty$;

4.feladat*

$a_n = (\sqrt{n^2+2n+3} - \sqrt{n^2+\alpha n+1})\cdot 1 = \frac{(n^2+2n+3)-(n^2+\alpha n+1)}{\sqrt{n^2+2n+3}+\sqrt{n^2+\alpha n+1}} = \frac{n}{n}\frac{(2-\alpha)+2/n}{\sqrt{1+2/n+3/n^2}+\sqrt{1+\alpha/n+1/n^2}} = \frac{(2-\alpha)+2/n}{\sqrt{1+2/n+3/n^2}+\sqrt{1+\alpha/n+1/n^2}}$

a, $\alpha \in \mathbb{R}$-re nem megoldható.

b, $\alpha = 2$

c, $\alpha \in \mathbb{R} \setminus \{2\}$

5.feladat*

$a_n = 6n^3 + 3 \to N(P) = (\frac{P-3}{6})^{\frac{1}{3}}$

$b_n = 6n^3 + 3n \geq 6n^3 + 3 \to N(P) = (\frac{P-3}{6})^{\frac{1}{3}}$

$c_n = \sqrt{n^2-n} = \sqrt{n(n-1)} \geq \sqrt{(n-1)^2} \to N(P) = \sqrt{P^2+1}$

$d_n = n^3 - 3n^2 + 5n + 9 \geq n^3 - 3n^2 = n^2(n-3) \overset{\text{ha } n\geq 3}{\geq} (n-3)^3 \to N(P) = max(P^{\frac{1}{3}}+3, 3)$

$f_n = \frac{n^3+3n}{n^2+2} \geq \frac{n^3}{n^2+2n^2} = \frac{n}{3} \to N(P) = 3P$