

Kalkulus 6. gyakorlat megoldások

2025. október 10.

1.feladat

a, $\lim_{n \rightarrow \infty} \frac{n^3 + 3n^2 - 7}{5n^3 + 9n} = \frac{1}{5}$

b, $\lim_{n \rightarrow \infty} \frac{3n^2 + 7n}{n^3 - 12n} = 0$

c, $\lim_{n \rightarrow \infty} \frac{n^3 + 3n^2 + 7}{3n^2 + 17} = \infty$

d, $\lim_{n \rightarrow \infty} \frac{n - 3n^2 - 1}{5 - n} = \infty$

e, $\lim_{n \rightarrow \infty} \frac{2n^2 - n^6 - n^5}{n - 4n^5 + n^8} = 0$

f, $\lim_{n \rightarrow \infty} \frac{n^4}{5 - 4\sqrt{n - 6n^4}} = -\frac{1}{6}$

2.feladat

a, $\lim_{n \rightarrow \infty} \sqrt{2n^2 + 5n} - \sqrt{2n^2 - 1} = \frac{3}{\sqrt{2}}$

b, $\lim_{n \rightarrow \infty} \sqrt{n^4 + 2n^2 + 3} - \sqrt{n^4 + n} = 1$

c, $\lim_{n \rightarrow \infty} \sqrt{4n^4 + n^2 - 2} - 2n^2 = \frac{1}{4}$

3.feladat

$a_n = n^5 - 7n^2 + 5n + 3 \geq n^5 - 7n^2 = n^2(n^3 - 7) \xrightarrow[\geq]{\text{ha } n^3 - 7 \geq 1 \text{ azaz } n \geq 2} n^2 \rightarrow \infty$

$b_n = \sqrt{n^5 + 2n^2} \geq \sqrt{n^5} = n^{\frac{5}{2}} \rightarrow \infty$

$c_n = \sqrt{n^5 - 2n^2} = \sqrt{n^2(n^3 - 2)} \geq \sqrt{n^2(n^2 - 2)} \xrightarrow[\geq]{\text{ha } n \geq 2} \sqrt{(n^2 - 2)(n^2 - 2)} = n^2 - 2 \rightarrow \infty$

$d_n = (\sqrt{n^4 + 2n^3} - \sqrt{n^4 + 5n^3}) \cdot 1 = \frac{(n^4 + 2n^3) - (n^4 + 5n^3)}{\sqrt{n^4 + 2n^3} + \sqrt{n^4 + 5n^3}} = \frac{n^3}{n^2} \frac{7}{\sqrt{1 + 2/n} + \sqrt{1 + 5/n}} = n \frac{7}{\sqrt{1 + 2/n} + \sqrt{1 + 5/n}} \rightarrow \infty$

4.feladat

$a_n = \frac{n!}{2^n} = \frac{n \cdot (n-1) \cdot (n-2) \dots 2 \cdot 1}{2 \cdot 2 \cdot 2 \dots 2 \cdot 2} \geq \frac{n}{2} \cdot 1 \cdot 1 \dots 1 \cdot \frac{1}{2} = \frac{n}{4} \rightarrow +\infty;$

$b_n = \frac{2^n}{n}: \frac{2^n}{n} > n, \text{ ha } n > 4, \text{ ezt teljes indukcióval lehet belátni, így a speciális rendő-elvből következik, hogy } b_n \rightarrow +\infty;$

5.feladat*

$a_n = (\sqrt{n^2 + 2n + 3} - \sqrt{n^2 + \alpha n + 1}) \cdot 1 = \frac{(n^2 + 2n + 3) - (n^2 + \alpha n + 1)}{\sqrt{n^2 + 2n + 3} + \sqrt{n^2 + \alpha n + 1}} = \frac{n}{n} \frac{(2 - \alpha) + 2/n}{\sqrt{1 + 2/n + 3/n^2} + \sqrt{1 + \alpha/n + 1/n^2}} = \frac{(2 - \alpha) + 2/n}{\sqrt{1 + 2/n + 3/n^2} + \sqrt{1 + \alpha/n + 1/n^2}}$

a, $\alpha \in \mathbb{R}$ -re nem megoldható.

b, $\alpha = 2$

c, $\alpha \in \mathbb{R} \setminus \{2\}$

6.feladat*

$$a_n = 6n^3 + 3 \rightarrow N(P) = \left(\frac{P-3}{6}\right)^{\frac{1}{3}}$$

$$b_n = 6n^3 + 3n \geq 6n^3 + 3 \rightarrow N(P) = \left(\frac{P-3}{6}\right)^{\frac{1}{3}}$$

$$c_n = \sqrt{n^2 - n} = \sqrt{n(n-1)} \geq \sqrt{(n-1)^2} \rightarrow N(P) = \sqrt{P^2 + 1}$$

$$d_n = n^3 - 3n^2 + 5n + 9 \geq n^3 - 3n^2 = n^2(n-3) \stackrel{\text{ha } n \geq 3}{\geq} (n-3)^3 \rightarrow N(P) = \max(P^{\frac{1}{3}} + 3, 3)$$

$$f_n = \frac{n^3+3n}{n^2+2} \geq \frac{n^3}{n^2+2n^2} = \frac{n}{3} \rightarrow N(P) = 3P$$