Kalkulus Minta Vizsga   2025 őA

BEUGRÓ:

1.a) $\left(\sqrt{\frac{4\cos x - 3^x}{\tan x + \log_2 x}}\right)' = ?$ komp. fv. $\quad x \xrightarrow{g} \frac{4\cos x - 3^x}{\tan x + \log_2 x} \xrightarrow{f} \sqrt{\dots}$

$$g(x) = \frac{4\cos x - 3^x}{\tan x + \log_2 x} \qquad \left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$$

$$g'(x) = \frac{\overbrace{(4\cos x - 3^x)'}^{-4\sin x - 3^x \cdot \ln 3} \cdot (\tan x + \log_2 x) - (4\cos x - 3^x)\overbrace{(\tan x + \log_2 x)'}^{\frac{1}{\cos^2 x} + \frac{1}{x \cdot \ln 2}}}{(\tan x + \log_2 x)^2}$$

$$f(x) = \sqrt{x} = x^{1/2} \qquad f'(x) = \frac{1}{2} \cdot x^{-1/2} = \frac{1}{2 \cdot \sqrt{x}}$$

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x) =$$

$$\frac{1}{2} \cdot \left(\frac{4\cos x - 3^x}{\tan x + \log_2 x}\right)^{-1/2} \cdot \frac{(-4\sin x - 3^x \cdot \ln 3)(\tan x + \log_2 x) - (4\cos x - 3^x) \cdot \left(\frac{1}{\cos^2 x} + \frac{1}{x \ln 2}\right)}{(\tan x + \log_2 x)^2}$$

1b)

$$\int_1^2 e^{2x-4} - 5^x - \frac{2}{3} \cdot \frac{1}{\sqrt{x}} - 2 \, dx \overset{\text{N.-L.}}{=} \left[\frac{e^{2x-4}}{2} - \frac{5^x}{\ln 5} - \frac{4}{3} x^{1/2} - 2x\right]_{x=1}^{2} \ominus$$

$$\int e^{2x-4} - 5^x - \frac{2}{3} \cdot x^{-1/2} - 2 \, dx = \frac{e^{2x-4}}{2} - \frac{5^x}{\ln 5} - \frac{2}{3} \cdot \frac{x^{1/2}}{1/2} - 2x + C$$

$$\int f(ax+b) = \frac{1}{a} F(ax+b) + c; \quad F = \int f. \qquad \int x^\alpha = \frac{x^{\alpha+1}}{\alpha+1} + c$$

$$\ominus \left(\frac{e^0}{2} - \frac{5^2}{\ln 5} - \frac{4}{3} 2^{1/2} - 4\right) - \left(\frac{e^{-2}}{2} - \frac{5}{\ln 5} - \frac{4}{3} \cdot 1 - 2\right)$$

1.c)

$$\int \frac{1}{x\cdot \ln x}\,dx = \int \frac{1/x}{\ln x}\,dx = \ln|\ln x| + C$$

$$\int \frac{f'}{f} = \ln|f| + C$$

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2)

$$\sqrt[n]{\frac{1}{3}}\cdot\sqrt[n]{n^2} = \sqrt[n]{\frac{2}{6}\cdot\frac{n^5}{n^3}} \le \sqrt[n]{\frac{2n^5}{5n^3+n^3}} \le \overbrace{\sqrt[n]{\frac{2n^5+4n+1}{5n^2+n^3}}}^{a_n} \le \sqrt[n]{\frac{2n^5+4n^5+n^5}{n^3}} = \sqrt[n]{\frac{7n^5}{n^3}} = \sqrt[n]{7}\cdot\sqrt[n]{n^2}$$

$\downarrow\ \downarrow$ (left): $1\cdot 1 = 1$

$\downarrow\ \downarrow$ (right): $1\cdot 1$, $\Downarrow$, 1

Rendőr-elv.

$\downarrow$

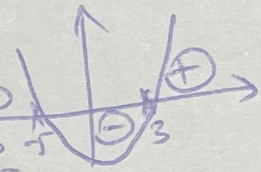
$\underline{\underline{1}}$

3)

$$\lim_{x\to 3}\frac{x-4}{x^2+2x-15} =$$

$x^2+2x-15=0$

$x_{1,2}=\frac{-2\pm\sqrt{64}}{2}$ 3, -5



Behely: $\frac{-1}{9+2\cdot 3-15} = \frac{-1}{0}$ $\boxed{\frac{\text{nem}}{0} = \infty}$ jobbról! balról!

jobbról:

$$\lim_{x\to 3+}\frac{x-4}{x^2+2x-15} = \frac{-1}{0^+} = \underline{\underline{-\infty}}$$

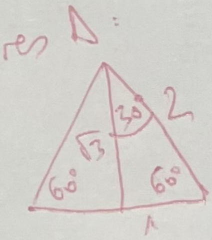
$$\lim_{x\to 3-}\frac{x-4}{x^2+2x-15} = \frac{-1}{0^-} = \underline{\underline{+\infty}}$$

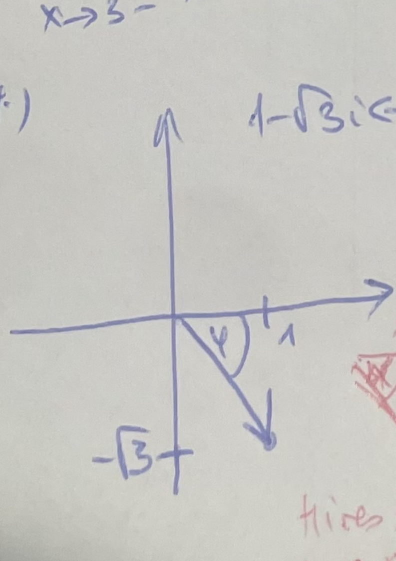
4.)

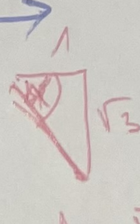
$1-\sqrt{3}i \Leftrightarrow (1,-\sqrt{3})$

$r = \sqrt{1^2+(-\sqrt{3})^2} = \sqrt{4} = \underline{\underline{2}}$

Híres Δ:







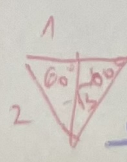
trigo alak:

$z = r\cdot(\cos\varphi + i\cdot\sin\varphi) =$

$= 2\cdot(\cos(-60°) + i\cdot\sin(-60°))$

Híres Δ

$\alpha = 60°$



$\rightarrow \varphi = \underline{\underline{-60°}}$

5.) $f(x)=\dfrac{x^2}{2x-8}$ $\quad D_f=\mathbb{R}\setminus\{4\}$ $\quad$ zh: $f(x)=0$ $\quad \dfrac{x^2}{2x-8}=0 \Leftrightarrow x^2=0 \Leftrightarrow \underline{x=0}$

$$f'(x)=\frac{2x\cdot(2x-8)-x^2\cdot 2}{(2x-8)^2}=\frac{4x^2-16x-2x^2}{(2x-8)^2}=\frac{2x^2-16x}{4(x-4)^2}=$$

$$f'(x)=\frac{x(x-8)}{2(x-4)^2}=\frac{x^2-8x}{2\cdot(x-4)^2}$$

$$f''(x)=\frac{(2x-8)\cdot 2\cdot(x-4)^2-(x^2-8x)\cdot 4\cdot(x-4)}{(2\cdot(x-4)^2)^2}=$$

egysz. $(x-4)$-el

$$=\frac{4(x-4)^3-4x(x-8)(x-4)}{4\cdot(x-4)^4}=\frac{\cancel{4}\cdot(x-4)^2-\cancel{4}x(x-8)}{\cancel{4}\cdot(x-4)^3}=\underline{\underline{\frac{16}{(x-4)^3}}}$$

$\Downarrow$

$f''(x)\neq 0 \Rightarrow$ nincs infl. pont

Áll:

f konvex $\Leftrightarrow f''\geq 0$ $\quad$ váltásnál infl. pont

f konkáv $\Leftrightarrow f''\leq 0$

	$(-\infty,4)$	$(4,+\infty)$
f''	$\ominus$	$\oplus$
f	$\cap$ konkáv	$\cup$ konvex

6) $\int e^{-x}(x-2)\,dx$ parc. integrál. $\int f\cdot g' = f\cdot g - \int f' g$

$$\int \underbrace{e^{-x}}_{g'(x)} \cdot \underbrace{(x-2)}_{f(x)}\,dx = -e^{-x}\cdot(x-2) - \int (-e^{-x})\cdot 1\,dx =$$

$g'(x)$	$f(x)$
$g(x) = \int e^{-x} = -e^{-x}$	$f'(x) = 1$

$$= -e^{-x}(x-2) + \underbrace{\int e^{-x}}_{-e^{-x}} =$$

$$= -e^{-x}(x-2) - e^{-x} + C =$$

$$= -e^{-x}(x-2+1) + C =$$

$$= -e^{-x}(x-1) + C$$

7)

$$\left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 6 & 4 & -2 & 5 \\ -1 & 10 & 3 & -7 \end{array}\right)$$

∴ Bokáros Gauss-elimináció!