Kalk pzh 2022. nov. 21.

KALKULUS MINTAZH 3 MEGOLDÁS

1) Det. Gauss-elim-vel:

$$\det A \underset{\det}{\Rightarrow} \begin{pmatrix} 2 & 2 & -6 \\ 4 & -1 & 7 \\ 5 & 0 & 4 \end{pmatrix} \sim 2\cdot \begin{vmatrix} 1 & 1 & -3 \\ 4 & -1 & 7 \\ 5 & 0 & 4 \end{vmatrix} \sim 2\cdot \begin{vmatrix} 1 & 1 & -3 \\ 0 & -5 & 19 \\ 0 & -5 & 19 \end{vmatrix} \sim$$

kiemelek I. sorból 2-t (2)

II − 4·I, III − 5·I (2)

III − II (2)

$$\sim 2\cdot \begin{vmatrix} 1 & 1 & -3 \\ 0 & -5 & 19 \\ 0 & 0 & 0 \end{vmatrix}$$

$\det A = 2\cdot 1\cdot(-5)\cdot 0 = 0$ (2)

$\text{rang}\,A = 2$ (5) → G. e. után nem 0 sorok száma

Dntx: det = főátlóbeli elemek szorzata (2)

$$A\cdot b \Rightarrow \begin{pmatrix} 2 & 2 & -6 \\ 4 & -1 & 7 \\ 5 & 0 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \begin{matrix} 1\cdot 2 + 2\cdot 2 + 3\cdot(-6) \\ 1\cdot 4 + (-1)\cdot 2 + 7\cdot 3 \\ 1\cdot 5 + 2\cdot 0 + 4\cdot 3 \end{matrix} \Rightarrow A\cdot b = \begin{pmatrix} 2+4-18 \\ 4-2+21 \\ 5+12 \end{pmatrix} = \begin{pmatrix} -12 \\ 23 \\ 17 \end{pmatrix}$$ (5)

(det A kifejtési tétellel is: (10))

2)

$$\underbrace{\sqrt[n]{2}}_{\to 1}^{\frac{1}{}} \cdot \underbrace{\sqrt[n]{n^2}}_{\to 1} = \sqrt[n]{\frac{n^4}{n^2+n^2}} \le \underbrace{\sqrt[n]{\frac{1+4n+n^4}{1+n^2}}}_{a_n} \le \sqrt[n]{\frac{n^4+4n^4+n^4}{n^2}} = \sqrt[n]{\frac{6n^4}{n^2}} = \underbrace{\sqrt[n]{6}}_{\to 1}\cdot\underbrace{\sqrt[n]{n^2}}_{\to 1}$$

(6) (6); dom: n^4, dom: n^2

$1\cdot 1 \to 1$ (3) $\qquad 1\cdot 1 \to 1$ (3)

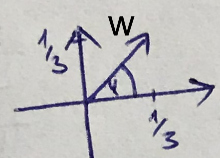
Rendőrelv: $a_n \to 1$. (2)

3) $z^3 = \frac{1+i}{3} = \underbrace{\frac{1}{3} + \frac{1}{3}i}_{\text{Alg. alak}} = w$ (1)

Alg. alak ⟶ Trig. alak

$|w| = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3} = r$ ez a w komplex szám hossza

w ábrázolva a síkon:



$\varphi = \frac{\pi}{4}$ ez a w komplex szám szöge

Trig. alak: $w = r\cdot(\cos\varphi + i\cdot\sin\varphi) = \frac{\sqrt{2}}{3}\left(\cos\left(\frac{\pi}{4}\right) + i\cdot\sin\left(\frac{\pi}{4}\right)\right)$ (4)

w-nek kell venni a harmadik gyökeit:

$$z_n = \sqrt[3]{\frac{\sqrt{2}}{3}}\cdot\left(\cos\left(\frac{\frac{\pi}{4}+n\cdot 2\pi}{3}\right) + i\cdot\sin\left(\frac{\frac{\pi}{4}+n\cdot 2\pi}{3}\right)\right)$$ (6)

Tehát

$$z_0 = \sqrt[3]{\frac{\sqrt{2}}{3}}\cdot\left(\cos\frac{\pi}{12} + i\cdot\sin\frac{\pi}{12}\right)$$ (2) $n = 0, 1, 2$

$$z_1 = \sqrt[3]{\frac{\sqrt{2}}{3}}\cdot\left(\cos\frac{9\pi}{12} + i\cdot\sin\frac{9\pi}{12}\right)$$ (2)

$$z_2 = \sqrt[3]{\frac{\sqrt{2}}{3}}\cdot\left(\cos\frac{17\pi}{12} + i\cdot\sin\frac{17\pi}{12}\right)$$ (2)

4.)

$$\lim_{x\to 1} \frac{x^2-1}{x^2+2x-3} = \lim_{x\to 1} \frac{(x-1)(x+1)}{(x-1)(x+3)} = \lim_{x\to 1} \frac{x+1}{x+3} = \frac{1+1}{1+3} = \frac{2}{4} = \frac{1}{2}$$

behely

(2)

behely: $\frac{1^2-1}{1^2+2-3} = \frac{0}{0}$ → gyöktényezőkre bont! (3)

(2)

$x^2-1=(x-1)(x+1)$

$x^2+2x-3=0$

$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{2}$ ↗ 1, ↘ -3

$x^2+2x-3=(x-1)(x+3)$ (4)

(2)

5. a) szorzat: $(fg)' = f'g + fg'$ (1)

$$\left[\left(\sqrt[3]{x}-x\right) sh(5x-3)\right]' = \left(\sqrt[3]{x}-x\right)' \cdot sh(5x-3) + \left(\sqrt[3]{x}-x\right)\left(sh(5x-3)\right)'$$

$$\left(\frac{1}{3}x^{-2/3}-1\right) \cdot sh(5x-3) + \left(\sqrt[3]{x}-x\right) ch(5x-3) \cdot 5$$ (1)

$$\left(\sqrt[3]{x}-x\right)' = \left(x^{1/3}-x\right)' = \frac{1}{3}x^{-2/3}-1$$ (2)

$[sh(5x-3)]' = ch(5x-3) \cdot 5$

összetett: $x \xrightarrow{g} 5x-3 \xrightarrow{f} sh(5x-3)$ (2)

$g(x)=5x-3 \quad g'(x)=5$

$f(x)=sh\,x \quad f'(x)=ch\,x$

$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

5. b)

$$\left(\frac{3^{x-1}-4}{\tan x}\right)' = \frac{(3^{x-1}-4)'\cdot \tan x - (3^{x-1}-4)(\tan x)'}{(\tan x)^2} \ (=) \quad ②$$

$$(=) \ \frac{3^{x-1}\cdot \ln 3\cdot 1\cdot \tan x - (3^{x-1}-4)\frac{1}{\cos^2 x}}{(\tan x)^2} \quad ②$$

kéyedbs:

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad ①$$

$$(3^{x-1}-4)' = (3^{x-1})' - \underbrace{(4)'}_{0} \quad ①$$

Eoypi: $x \overset{g}{\longmapsto} x-1 \overset{f}{\longmapsto} 3^{x-1}$

$g(x) = x-1 \quad g'(x) = 1$

$f(x) = 3^x \quad f'(x) = 3^x\cdot \ln 3$

$(f\circ g)'(x) = f'(g(x))\cdot g'(x) = 3^{x-1}\cdot \ln 3\cdot 1$ ④

$$(\tan x)' = \frac{1}{\cos^2 x} \quad ②$$