Kalkulus vizsga 4. Mo

2021. jún.

1.) (15p) A kibővített eh. mtx:

$$\left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 7 & 3 & 6 & -9 \\ 8 & 5 & 11 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & -11 & -29 & -9 \\ 0 & -11 & -29 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & -11 & -29 & -9 \\ 0 & 0 & 0 & 12 \end{array}\right)$$

$II - 7I$ (3)

$III - 8I$ (3)

$III - II$ (3)

tilos sor (3)

$\Downarrow$

A LER-nek nincs mo-a (3)

2.) (15p)

$$\lim_{n\to+\infty}\left(1+\frac{2}{3n+1}\right)^{3n+4} = \lim\left(\frac{3n+3}{3n+1}\right)^{3n+4} = \text{(1)}$$

$$= \lim \underbrace{\left(\frac{3n+3}{3n+1}\right)^{3n}}_{\text{(3)}} \cdot \underbrace{\left(\frac{3n+3}{3n+1}\right)^{4}} = \underline{\underline{e^2}} \text{ (1)}$$

$$\text{(3)} \left\{ \frac{\left(1+\frac{3}{3n}\right)^{3n} \to e^3 \text{ (2)}}{\left(1+\frac{1}{3n}\right)^{3n} \to e^1 \text{ (1)}} \right. \qquad \left(\frac{1+\frac{1}{n}}{1+\frac{1}{3n}}\right)^4$$

$$\downarrow \qquad\qquad \downarrow \text{ (3)}$$

$$\frac{e^3}{e^1} = e^2 \text{ (1)} \qquad 1^4 = 1$$

VAGY:

$$\lim \underbrace{\left(1+\frac{2}{3n+1}\right)^{3n+1}}_{\text{(7)}} \left(1+\frac{2}{3n+1}\right)^{3} = \underline{\underline{e^2}} \text{ (1)}$$

$$\downarrow e^2 \qquad\qquad \downarrow \text{ (7)} \; 1$$

indoklás ...

3.) 15p

$$\lim_{x\to 2+} \frac{x^3-2x^2}{|x-2|} \overset{\text{"}\frac{0}{0}\text{"}\ 1}{=} \lim_{x\to 2+} \frac{x^2 \cancel{(x-2)}}{\cancel{x-2}} \;1 = \lim_{x\to 2+} x^2 = 2^2 = \underline{\underline{4}} \;1$$

$x\to 2+ \Rightarrow x>2$

$|x-2| = x-2$ ha $x>2$ } 2

$$\lim_{x\to 2-} \frac{x^3-2x^2}{|x-2|} = \lim_{x\to 2-} \frac{x^2\cdot(x-2)}{2-x} \;1 = \lim_{x\to 2-} (-x^2) = \underline{\underline{-4}} \;1$$

$x\to 2- \Rightarrow x<2$

$|x-2| = 2-x$ ha $x<2$ } 2

$\lim_{x\to 2} \frac{x^3-2x^2}{|x-2|} \nexists$ 1, mi. a jobboldali h.é. $\neq$ 2 bal o. i hat. é.

4.) 10p Ez egy összetett fv. 1 $(f\circ g)(x)$

$$\begin{cases} f(x) = \sqrt[3]{x} = x^{1/3} \Rightarrow f'(x) = \frac{1}{3}\cdot x^{-2/3} \;1 \\ g(x) = \frac{e^{2x}}{\cosh(x)+3x^2} \quad \text{ez egy hányados fv: } 1 \end{cases} \;1$$

$$g'(x) = \frac{\overbrace{(e^{2x})'}^{\text{ez is komp. fv } 1}\cdot(\cosh(x)+3x^2) - e^{2x}\cdot\overbrace{(\cosh(x)+3x^2)'}^{\text{összegfv. } 1}}{(\cosh(x)+3x^2)^2} =$$

$$= \frac{2\cdot e^{2x}\cdot(\cosh(x)+3x^2) - e^{2x}\cdot(\sinh(x)+6x)}{(\cosh(x)+3x^2)^2} \Bigg\} 2$$

<u>Tehát:</u>

$$(f\circ g)'(x) = f'(g(x))\cdot g'(x) \;1 = \frac{1}{3}\cdot\left(\frac{e^{2x}}{\cosh(x)+3x^2}\right)^{-2/3}\cdot \left(\frac{2e^{2x}(\cosh(x)+3x^2) - e^{2x}\cdot(\sinh(x)+6x)}{(\cosh(x)+3x^2)^2}\right) \;1$$

5.) (10p) Tétel f kétszer diffható, így egy I int.-ra

f konvex I-n $\iff$ $f'' \geq 0$ I-n,

f konkáv I-n $\iff$ $f'' \leq 0$ I-n. (2)

f-nek x_0 infl. pontja $\iff$ f'' előjelet vált x_0-ban (2)

$f(x) = x^4 + 14x^3 - 180x^2$ $\quad D_f = \mathbb{R}$ (1)

$f'(x) = 4x^3 + 42x^2 - 360x$ (2)

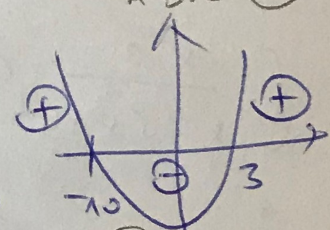
$f''(x) = 12x^2 + 84x - 360$ (2)

$f''(x) = 12(x^2 + 7x - 30) = 0$ (2)

$x^2 + 7x - 30 = 0$

$x_{1,2} = \dfrac{-7 \pm \sqrt{49 + 120}}{2}$ $\begin{matrix} 3 \\ -10 \end{matrix}$ (2)

Ábra: (2)



	$(-\infty, -10)$	$x = -10$	$(-10, 3)$	$x = 3$	$(3, +\infty)$
f''	+	0	−	0	+
f	konvex	infl. p.	konkáv	infl. p.	konvex

(5)

6.a) 15p parc. törtekre bontás ②

$$\frac{x-3}{x^2-5x+4} \stackrel{!}{=} \frac{A}{x-1} + \frac{B}{x-4} = \frac{Ax-4A+Bx-B}{(x-1)\cdot(x-4)} \quad ②$$

$$\Updownarrow$$

$x^2-5x+4=0$

$x_{1,2} = \ldots$ 1, 4

$$x-3 = (A+B)x - 4A - B \quad ②$$

$$\Updownarrow$$

$$② \begin{cases} 1 = A+B \\ -3 = -4A-B \end{cases} \Rightarrow B = 1-A$$

$$② \begin{cases} -3 = -4A-(1-A) = -3A-1 \\ 3A = 2 \\ A = \frac{2}{3} \end{cases} \Rightarrow B = 1-A = 1-\frac{2}{3} = \frac{1}{3}$$

Tehát

$$\int \frac{x-3}{x^2-5x+4}\,dx = \int \overbrace{\frac{2}{3}\cdot\frac{1}{x-1} + \frac{1}{3}\cdot\frac{1}{x-4}}^{②}\,dx = \overbrace{\frac{2}{3}\ln|x-1| + \frac{1}{3}\ln|x-4|}^{②} + C$$

$C \in \mathbb{R}$ tetsz.

① $\int \frac{1}{ax+b} = \frac{1}{a}\cdot\ln|ax+b| + C$

6. b)

$$\int_0^{\pi} \sin 2x\,dx = \overbrace{\left[\frac{-\cos 2x}{2}\right]_{x=0}^{\pi}}^{④} = \overbrace{-\frac{\cos 2\pi}{2} + \frac{\cos 0}{2}}^{②} = \overbrace{-\frac{1}{2}+\frac{1}{2}}^{①} = \underline{\underline{0}} \quad ①$$

② $\int f(ax+b) = \frac{1}{a}F(ax+b)$, $\int f = F$

7. 15p $y = f'(x_0)\cdot(x-x_0) + f(x_0)$ érintő egyenlete ① $x_0 = 0$

$f(x) = e^{-3x} + x$ $\qquad f(x_0) = f(0) = e^0 = 1$ ③

$f'(x) = -3\cdot e^{-3x} + 1$ ④ $\qquad f'(x_0) = f'(0) = -3+1 = -2$ ③

komp. der!

$e:\quad y = -2\cdot(x-0)+1$ ②

$\boxed{y = -2x+1}$ ②