Kalkulus vizsga 1

2021. dec. 22.

1.) (15p)

$$\lim_{n\to+\infty}\left(\overbrace{\sqrt{n^3+3n}}^{A}-\overbrace{\sqrt{n^3-5n^2}}^{B}\right) \Rightarrow$$

$$(A-B)\cdot\frac{A+B}{A+B}=\frac{A^2-B^2}{A+B}$$

$$\lim_{n\to+\infty}\frac{\overbrace{(n^3+3n)-(n^3-5n^2)}^{+5n^2+3n \text{ dom } (1)}\;(4)}{\underbrace{\sqrt{n^3+3n}}_{\sqrt{n^3(1+\frac{3}{n^2})} = n^{3/2}\cdot\sqrt{1+\frac{3}{n^2}}}+\underbrace{\sqrt{n^3-5n^2}}_{\sqrt{n^3(1-\frac{5}{n})} = n^{3/2}\cdot\sqrt{1-\frac{5}{n}}}}=\lim_{n\to+\infty}\overbrace{\boxed{\frac{n^2}{n^{3/2}}}}^{n^{1/2}\;(2)}\cdot\frac{\overbrace{5+\frac{3}{n}}^{(2)}}{\sqrt{1+\frac{3}{n^2}}+\sqrt{1-\frac{5}{n}}} \ominus$$

dom (1) dom (1)

$$\ominus \lim_{n\to+\infty} \underbrace{n^{1/2}}_{\downarrow\, +\infty\; (1)}\cdot\frac{5+\frac{3}{n}}{\sqrt{1+\frac{3}{n^2}}+\sqrt{1+\frac{5}{n}}}\;(1)=+\infty\cdot\frac{5}{\sqrt{1}+\sqrt{1}}=+\infty\cdot\frac{5}{2}=+\infty \;(1)$$

$\frac{3}{n}\to 0$, $\frac{3}{n^2}\to 0$, $\frac{5}{n}\to 0$ (1)

2.) (15p)

$$\lim_{x\to 0}\frac{\cos(2x^3)-1}{6x^3}=\lim_{x\to 0}\frac{\overbrace{-\sin(2x^3)\cdot\cancel{6x^2}}^{(5)}}{\cancel{18x^2}}=\lim_{x\to 0}\left(-\frac{1}{3}\right)\cdot\sin(2x^3)=$$

$$\frac{\cos 0-1}{0}=\frac{0}{0}\to L'H \;(5)$$

$$-\frac{1}{3}\cdot\underbrace{\sin 0}_{0}=\underline{\underline{0}} \;(5)$$

VAGY

$$\lim_{x\to 0}\frac{\cos(2x^3)-1}{6x^3}=\lim_{x\to 0}\frac{\cos(2x^3)-1}{6x^3}\cdot\frac{\cos(2x^3)+1}{\cos(2x^3)+1}\;(3)=$$

$\frac{0}{0}$:(

$$=\lim_{x\to 0}\frac{\cos^2(2x^3)-1}{6x^3\cdot\underbrace{(\cos(2x^3)+1)}_{\to\cos 0+1=2\;(1)}}\;(2)=\lim_{x\to 0}\frac{-\sin^2(2x^3)}{3\cdot 2x^3\cdot 2}\;(2)=\lim_{x\to 0}\left(-\frac{1}{6}\right)\cdot\underbrace{\frac{\sin(2x^3)}{2x^3}}_{\downarrow\,1\;(1)}\;(2)\cdot\underbrace{\sin(2x^3)}_{\downarrow\,\sin 0 = 0\;(1)}\;(1)$$

$\frac{\sin\alpha}{\alpha}\to 1$ ha $\alpha\to 0$

$$\ominus\left(-\frac{1}{6}\right)\cdot 1\cdot 0=\underline{\underline{0}}\;(1)$$

3.) (15p) $z^2 - \bar{z}^2 + z = 1+5i$

Legyen $z = a+ib$ (1) $\Rightarrow$ $\bar{z} = a-ib$ (1)

$(a+ib)^2 - (a-ib)^2 + a+ib = 1+5i$ (2)

$a^2 + 2abi + \underbrace{i^2}_{-1} b^2 - a^2 + 2abi - \underbrace{i^2}_{-1} b^2 + a+ib = 1+5i$ (2)

$4abi + i\cdot b + a = 1+5i$ (2)

Valós rész: $a = 1$ (2)

Képzetes rész: $4ab + b = 5$ (2)

$4b + b = 5$

$5b = 5$

$b = 1$ (2)

Tehát

$z = a+ib = 1+i$ (1)

4.) (15p) Negyedfokú T-poli: (3)

$T_4(x) = f(x_0) + f'(x_0)\cdot(x-x_0) + \frac{f''(x_0)}{2!}\cdot(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^2 + \frac{f^{(4)}(x_0)}{4!}(x-x_0)^4$

$f(x) - \sin(-2x) - x$ $\quad x_0 = 0$ $\quad f(0) = \sin 0 - 0 = 0$ (1)

$f'(x) = \cos(-2x)\cdot(-2) - 1$ (1) $\quad f'(0) = \overset{1}{\cos 0}\cdot(-2) - 1 = -2-1 = -3$ (1)

$f''(x) = (-2)\cdot(-2)\cdot(-\sin(-2x))$ (1) $\quad f''(0) = -4\cdot\sin 0 = 0$ (1)

$f'''(x) = (-2)^3\cdot(-\cos(-2x))$ (1) $\quad f'''(0) = -8\cdot(-\cos 0) = 8$ (1)

$f^{(4)}(x) = (-2)^4\cdot(\sin(-2x))$ (1) $\quad f^{(4)}(0) = (-2)^4\cdot\sin 0 = 0$ (1)

$T_4(x) = 0 + (-3)\cdot x + 0 + \frac{8}{3!}\cdot x^3 + 0 = -3x + \frac{4}{3}x^3$ (3)

5.) a) (10p)

$$\int \frac{1}{8x^2+2}\,dx = \frac{1}{2}\int \frac{1}{(2x)^2+1}\,dx = \frac{1}{2}\cdot\frac{\operatorname{arctg}(2x)}{2} + \cancel{c}$$

(1) (3) (arctg 0 = 0)

$D<0$

(2) Lin. hely. $\int_0^{1/2} \frac{1}{8x^2+2} = \frac{1}{4}\cdot\left[\operatorname{arctg}(2x)\right]_{x=0}^{1/2} = \frac{1}{4}\operatorname{arctg}1 = \frac{\pi}{16}$ (2) (1)

N.L. (1)

b) (10p)

$$\int_0^{3\pi}(2\cos(x))^8 \sin(x)\,dx = 2^8\cdot\int_0^{3\pi}\cos^8 x\cdot\sin x = -256\underbrace{\int_0^{3\pi}(\cos x)^8\cdot(-\sin x)}_{1} \;=$$

(1)

$\int \cos^8 x\cdot \sin x$

(2)

$$\int f^\alpha\cdot f' = \int \cos^8 x\cdot(-\sin x) = \frac{\cos^9 x}{9} + c = \frac{f^{\alpha+1}}{\alpha+1}+c$$

$f(x)=\cos x$ $\quad \alpha = 8$

$f'(x) = -\sin x$

$$= \underbrace{-256\cdot\left[\frac{(\cos x)^9}{9}\right]_{x=0}^{3\pi}}_{2} = -\frac{256}{9}\cdot\underbrace{\left(\underbrace{(\cos 3\pi)^9}_{(-1)^9} - \underbrace{(\cos 0)^9}_{1^9}\right)}_{2} =$$

N.L.

$$-\frac{256}{9}\cdot(-1-1) = \frac{2\cdot 256}{9} = \frac{512}{9}$$ (2)

6.) (20p)

$$\int \frac{5}{x^2+3x-4}\,dx = \int \frac{1}{x-1} - \frac{1}{x+4} = \ln|x-1| - \ln|x+4| =$$ (2)

$$= \ln\left|\frac{x-1}{x+4}\right|$$ (2)

$$x_{1,2} = \frac{-3\pm\sqrt{9+16}}{2} \;\begin{matrix} \nearrow 1 \\ \searrow -4 \end{matrix}$$ (3)

parc. törtekre: (1)

$$\frac{5}{x^2+3x-4} = \frac{A}{x-1} + \frac{B}{x+4} = \frac{1}{x-1} + \frac{-1}{x+4}$$ (4)

$5 = A(x+4) + B(x-1)$

$x=-4 \Rightarrow 5 = B\cdot(-5) \Rightarrow B=-1$

$x = 1 \Rightarrow 5 = A\cdot 5 \Rightarrow A = 1$

$\ln 1 = 0$ (1)

$$\int_2^{+\infty}\frac{5}{x^2+3x-4} = \lim_{N\to+\infty}\left[\ln\left|\frac{x-1}{x+4}\right|\right]_{x=2}^{N} = \lim_{N\to+\infty}\ln\left|\frac{N-1}{N+4}\right| - \ln\frac{1}{6} = -\ln\frac{1}{6}$$

(2) (2) (2)

$= \ln 6$ (1)

f) (15 p)

$$\left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 6 & 4 & -2 & 5 \\ -1 & 10 & 3 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 0 & 4 & 1 & 5 \\ 0 & 10 & \frac{5}{2} & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & -1 & 0 \\ 0 & 4 & 1 & 5 \\ 0 & 0 & 0 & -\frac{39}{2} \end{array}\right)$$

$II - 3 \cdot I$ (4)
$III + \frac{1}{2} I$ (4)

$III - \frac{5}{2} \cdot II$ (4)

tilos sor
$\Downarrow$
nincs megoldás!

(3)