BSM Course on Markov Chains and Dynamical Systems, Spring 2025

Quiz #7, April 25; **NAME:**

1. Consider the map $T : [0,1] \to [0,1]$,

$$T(x) = \begin{cases} \frac{3}{2}x & \text{if } 0 \le x < \frac{2}{3}, \\ 3x - 2 & \text{if } \frac{2}{3} \le x \le 1. \end{cases}$$

Is Lebesgue measure invariant for T? If yes, explain why, if no, find another absolutely continuous invariant measure (ie. an invariant density).

2. Consider the map $T_\lambda : \mathbb{R} \to \mathbb{R}$, $T_\lambda(x) = x^2 + \lambda$, specifically for (a) $\lambda = 0$ and (b) $\lambda = 2$. For both cases, sketch the graph of T_λ, find its fixed points and determine their stability (attracting or repelling). Is there some $\lambda \in (0,2)$ that can be considered as a bifurcation value? If yes, for what type of bifurcation, and why?