# Markov Chains and Dynamical Systems, Spring 2025

## Homework problem set #3 . Due on March 28, Friday

Problems 1.-3. are from Durrett, R.: Essentials of Stochastic Processes (Section 1.12), available at the author's webpage.

1. Problem 1.59.
2. Problem 1.70.
3. Problem 1.71.
4. Problem 1.75. In the positive recurrent case, compute the stationary distribution for the chain. (Note that here $p_n = p(0, n)$ make a probability distribution of some random variable $\xi$, which gives where the chain jumps from the state 0. Recall from class that $\mathbb{E}\xi = \sum_{n\geq 1} \mathbb{P}(\xi \geq n)$.)
5. Problem 1.77. (Note Example 7.2 is a typo, it is supposed to be Example 1.8.)
6. Consider a computer program which processes exactly 1 computational request per time unit. Within each time unit, new requests arrive, and thus put at the end of a queue. The number of new arrivals per time unit is random and has the following distribution:

| $k$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $\mathbb{P}(k$ requests per time unit$)$ | 0.4 | $p$ | $0.5 - p$ | 0.1 |

   where $p \in (0, 0.5)$ is some parameter. Let us assume that initially the queue is empty, and at time 0 the zeroth request arrives. This request is processed by time 1, and let us refer to the requests that arrive while this zeroth request is processed as the first generation of requests. The number of the first generation requests is a random variable which we denote by $X_1$. Proceed inductively and define, for $n \geq 2$, the „$n$th generation of requests" as the requests that arrive while the $(n-1)$st generation of requests is processed. Let, furthermore, $X_n$ denote the cardinality of the $n$th generation of requests. Answer the questions below for (i) $p = 0.2$ and (ii) $p = 0.4$.

   (a) Determine the expected value and the generating function of $X_1$.

   (b) Determine the generating function of $X_2$.

   (c) $\mathbb{E}(X_{72}) =?$

   (d) Compute $r_3 = \mathbb{P}(X_3 = 0)$.

   (e) What is the probability that (sooner or later) we will have an empty queue? (Hint: It is not hard to solve a cubic equation if you know one of its roots.)