BSM Course on Markov Chains and Dynamical Systems, Spring 2026

Quiz #6, April 14; **NAME:**

1. The questions below concern the *doubling map*, that is $T : \mathbb{S}^1 \to \mathbb{S}^1$, $Tx = 2x \, (mod\, 1)$.

   (a) What is the orbit of $x = \frac{6}{7}$ under T?

   (b) Find the binary code of $x = \frac{6}{7}$, that is, the sequence of digits $x_k \in \{0, 1\}$, $k = 1, 2, \ldots$ such that $\frac{6}{7} = \sum_{k=1}^{\infty} x_k 2^{-k}$.

   (c) Find some $y \in \mathbb{S}^1$ such that (i) $|y - \frac{6}{7}| \leq \frac{1}{32}$, (ii) y is irrational, (iii) the orbit of y is *not* dense on $\mathbb{S}^1$.

2. (a) Sketch the graph of the function $G : \mathbb{R}^+ \to \mathbb{R}^+$, $G(x) = \dfrac{2}{3}\, x + \dfrac{3}{x}$.

   (b) Consider the recursion $x_0 = 200$, $x_n = G(x_{n-1}) = \dfrac{2}{3}\, x_{n-1} + \dfrac{3}{x_{n-1}}$. $\lim\limits_{n\to\infty} x_n = ?$