

Consider the 2×2 matrices $A = A_1$ and $A = A_2$. For both of them:

(a) Sketch the phase portrait of $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $S(\underline{w}) = A \cdot \underline{w}$.

(b) Decide if (and argue why) it is possible to define a toral automorphism $T : \mathbb{T}^2 \rightarrow \mathbb{T}^2$, $T(\underline{w}) = A \cdot \underline{w} \pmod{\mathbb{Z}^2}$.

If yes, decide if this toral automorphism is hyperbolic.

1. $A_1 = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$.

2. $A_2 = \begin{pmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{pmatrix}$.