

1. The questions below concern the *doubling map*, that is  $T : \mathbb{S}^1 \rightarrow \mathbb{S}^1$ ,  $Tx = 2x \pmod{1}$ .

(a) What is the orbit of  $x = \frac{6}{7}$  under  $T$ ?

(b) Find the binary code of  $x = \frac{6}{7}$ , that is, the sequence of digits  $x_k \in \{0, 1\}$ ,  $k = 1, 2, \dots$  such that  $\frac{6}{7} = \sum_{k=1}^{\infty} x_k 2^{-k}$ .

(c) Find some  $y \in \mathbb{S}^1$  such that (i)  $|y - \frac{6}{7}| \leq \frac{1}{32}$ , (ii)  $y$  is irrational, (iii) the orbit of  $y$  is *not* dense on  $\mathbb{S}^1$ .

2. (a) Sketch the graph of the function  $G : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ ,  $G(x) = \frac{2}{3}x + \frac{3}{x}$ .

(b) Consider the recursion  $x_0 = 200$ ,  $x_n = G(x_{n-1}) = \frac{2}{3}x_{n-1} + \frac{3}{x_{n-1}}$ .  $\lim_{n \rightarrow \infty} x_n = ?$