PROBABILITY SYLLABUS, Spring Semester 2020
Budapest Semesters in Mathematics
Tu 10:15am - noon, Room 002 (until March 12)
Th 8:15am - 10:00, Room 002 (until March 12)

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Text is: *A First Course in Probability* by S. Ross
(preferably Eighth Edition or later, but earlier editions also work)
Course homepage: `www.math.bme.hu/~pet/pro/`

Update on online teaching, from March 23

Below you find a summary on the main features of the online continuation of the course. The content of this syllabus is continuously updated, check back regularly.

- Various documents will be posted in *piazza's resources section* for self study. In particular
  - the continuously updated *syllabus* containing the *schedule* for the zoom meetings and *the homework assignments* with due dates.
  - *detailed and continuously updated notes* in which the class material is discussed, with references to the relevant sections of the Ross book, augmented with explanations, and further supplementary material.
- Given that the semester officially ends on April 17 (with exams on the week of April 20)
  - we will try to proceed in a slightly faster pace than originally planned, so that we have a chance to cover 75% of the material by April 17. Yet, I am happy to continue after April 17 if you are interested!
  - instead of two midterm exams, there is just one midterm exam this semester (the one you have already taken). Your grades will be based on homework solutions, the midterm exam and the final exam.
  - midterm assessment grades will be uploaded after I have graded Homeworks 5 and 6 (due on March 31). Note that this is just a snapshot and it does not have strong implications towards your final grade.
- Our main means of communication are
  - two *one-hour-long zoom meetings weekly*, to be scheduled.
  - the *Q&A section of piazza*. I encourage you to post your questions here well before the zoom sessions, to save time.
  - further online consultation sessions, for the whole class, or individually (office hour style), can be organized upon request.
- The first week online (March 23–27) will focus on
  - new material: sections 4.7, 4.8, 4.9, 4.10, 5.1, 5.2 and 5.3 in the Ross book. Further resources to be posted soon!
  - consultation on Homeworks 5 and 6.
- further details to follow. I am looking forward to a great online probability experience with you!

Course Description: This is a first course on the mathematical phenomenon of uncertainty and techniques used to handle them. Not only being challenging itself, this field is of increasing interest in many areas of engineering, economical, physical, biological and sociological sciences as well. In this course we cover the basic notions and methods of probability theory, also giving emphasize on examples, applications and problem solving. Briefly, the topics include probability in discrete sample spaces, methods of enumeration (combinatorics), conditional probability and independence, random variables, properties of expectations, the Weak Law of Large Numbers, and the Central Limit Theorem.

Probability is a conceptually difficult field, although it might seem easy and straightforward at first. One has to distinguish between very different mathematical objects, and find their connection to real-life situations within the same problem. Therefore it is very important to follow classes and deeply understand the material during the semester.

Grading and assignments: There will be two in-class exams, weekly homeworks to be handed in during the semester, and a final exam.

- **Two in-class exams** are to be scheduled in due time. The first exam will be on Chapters 1, 2 and 3, the second one will be on Chapters 4, 5 and 6. **Each** worth **160 points** (each 20% of the total possible points).
- **13 homework sets** are to be handed in during the semester. **Each** worth **20 points**, the worst of all homeworks will be dropped. This way, a **total** of **240 points** (30% of the total possible points) can be earned from these assignments. Solving the homework problems by no means guarantees that you have the necessary level of practice. Please do other exercises (and check the answers in the back of the book *after* solving them) until you feel safe with problems on the topics in question. It is a good idea to simulate exam-like situations: solve exercises in limited time, without the use of the book or your notes (or your classmates).
- **The final exam** is to be scheduled in due time. Half of it will cover Chapters 1 to 6, the other half is on Chapters 7 and 8 of the book. It is worth **240 points** (30 % of the total possible points).
- **Bonus questions** are also to be found in the homework sets. While a total of 800 points can be earned by the exams and homeworks, an additional **4 points** can be given for a solution of each bonus problem.

Grades will be based on the total of 800 points approximating the following standards:

Grade	**Points**
A^+	≥ 775
A	$\in [745, 775)$
A^-	$\in [720, 745)$
B^+	$\in [695, 720)$
B	$\in [665, 695)$
B^-	$\in [640, 665)$
C^+	$\in [615, 640)$
C	$\in [585, 615)$
C^-	$\in [560, 585)$
D	$\in [480, 560)$
F	< 480

Because of this standard, you are not in competition with your classmates nor does their performance influence positively or negatively your performance. You are encouraged to form study/problem groups with your classmates; things not clear to you may become obvious when you try to explain them to others or when you hear other points of view. Sometimes just verbalizing your mathematical thoughts can deepen your understanding. However, if you discuss with others the exercises, each person should write up her/his own version of the solution. Please note that much less can be learned by just understanding and writing up someone else's solution than by coming up (or even just trying to come up) with original ideas and solving the problem.

Please feel free to contact me any time outside class via e-mail, phone, or in person if you have questions or suggestions about this course.

A tentative course schedule for the first couple of weeks with the already assigned homework problem sets follows on the next pages. The content of the syllabus is continuously updated as more information are available.

Tentative **Schedule for BSM Probability, Spring 2020**

Date	Chapter of the book	Homework due on the date to the left
Feb 4 Tu	1.1, 1.2, 1.3, 1.4	-
Feb 6 Th	1.5, 1.6, 2.2, 2.3, 2.4	-
Feb 11 Tu	2.4, 2.5	-
Feb 13 Th	2.5, 2.6, 3.2	Homework #1
Feb 17 Mo, 4.30pm	**MUC, Room 206**, 3.2, 3.3	-
Feb 18 Tu	*Canceled*	-
Feb 20 Th	3.3, 3.4	Homework #2
Feb 25 Tu	3.4, 3.5	-
Feb 27 Th	3.5, 4.1, 4.2, 4.3	Homework #3
Mar 3 Tu	4.3, 4.4	-
Mar 5 Th	4.4, 4.5, 4.6	Homework #4
Mar 9 Mo, 4.30pm	**Consultation, Room 206**	-
Mar 10 Tu	**Midterm 1**, 4.7	-
Mar 12 – Mar 20	*Break*	-
...	...	
Mar 31 Tu	...	Homework #5, Homework #6

Starting March 23, the course continues in the form of distance education. Electronic submission of Homeworks #5 and #6 is due on March 31.

Homework problem sets for BSM Probability, Fall 2020

Group work is encouraged, but write up your own solution. Please *show your work leading to the result, not only the result.* Problems are either from the book or written here explicitly. Please make sure you solve the problem indicated here, and not another one (the one below or above it, or the problem with the same number but from another chapter or from the other edition, etc.). *Numbers refer to the 8th edition of the book. If you have another edition, let me know to check what the relevant numbers are.* Each problem worth the number of $^\bullet$'s you see right next to it. Hence, for example, Problem 4 of Chapter 1 worth two points, while Theoretical Exercise 10 of Chapter 1 worth three points. Bonus problems are also to be found here, each worth 4 points. Note that they are also due on the due date of the corresponding homework.

Homework #1 (Due on February 13)

Chapter 1, Problem $4^{\bullet\bullet}$, $7^{\bullet\bullet\bullet}$, $15^{\bullet\bullet\bullet}$, $19^{\bullet\bullet\bullet}$, $21^{\bullet\bullet\bullet}$, $31^{\bullet\bullet\bullet}$,
Chapter 1, Theoretical Exercise $10^{\bullet\bullet\bullet}$

Homework #2 (Due on February 20)

Chapter 2, Problem $13^{\bullet\bullet}$, $17^{\bullet\bullet}$, $21^{\bullet\bullet}$, $23^{\bullet\bullet}$, $27^{\bullet\bullet}$, $33^{\bullet\bullet}$, $44^{\bullet\bullet}$,
Problem #2A$^{\bullet\bullet\bullet\bullet}$: We roll a die ten times. What is the probability that each of the results $1, 2, \ldots, 6$ shows up at least once? HINT: Define the events $A_i := \{$number i doesn't show up at all during the ten rolls$\}$, $i = 1 \ldots 6$. Note that these events are *not* mutually exclusive.
Problem #2B$^{\bullet\bullet}$: For the events A and B, we know that $P(A) \geq 0.8$ and $P(B) \geq 0.5$. Show that $P(A \cap B) \geq 0.3$.
Bonus problem #2C: A closet contains n pairs of shoes. If $2r$ shoes are randomly selected ($2r \leq n$), what is the probability that there will be (a) no complete pair, (b) exactly one complete pair, (c) exactly two complete pairs?

Homework #3 (Due on February 27)

Chapter 3, Problem 7$^{\bullet\bullet\bullet}$ (HINT: it is not $\frac{1}{2}$.), 12$^{\bullet\bullet\bullet\bullet}$, 44$^{\bullet\bullet\bullet}$, 50$^{\bullet\bullet\bullet}$,
Problem #3A$^{\bullet\bullet\bullet}$: We repeatedly roll two dice at the same time, and only stop when at least one of them shows a six. What is the probability that the other also shows a six? (HINT: it is not $\frac{1}{6}$).
Problem #3B$^{\bullet\bullet\bullet\bullet}$: Eggs are sold in boxes containing 10 eggs at the farmer's market. In 60% of these boxes all the eggs are unbroken, in 30% of the boxes there is exactly one broken egg, in 10% of the boxes there are exactly two broken eggs. I buy a box of eggs at the farmer's market, at home I take an egg out of it and I realize that it is broken. What is the chance that there is another broken egg in this box?
Bonus problem #3C: Consider a group of n people, and let A_{ij} denote the event that person $\#i$ and person $\#j$ have a common birthday. (a) Are these $\binom{n}{2}$ events *pairwise* independent? (b) Are these $\binom{n}{2}$ events independent as a collection?

Homework #4 (Due on March 5)

Chapter 3, Problem 74$^{\bullet\bullet\bullet}$; Theoretical Exercise 22$^{\bullet\bullet\bullet}$;
Problem #4A$^{\bullet\bullet\bullet}$: Die α has four red and two white faces, while die β has two red and four white faces. We flip a fair coin. If it comes head then we use die α, if it comes tail, then we use die β. We then roll the die selected this way n times. What is the probability that the k-th roll will be red, given that all previous rolls were red ($k = 1, 2, \ldots, n$)?
Problem #4B$^{\bullet\bullet}$: Andrew and Bob play the following game. There are 5 red balls and 5 blue balls in a urn, out of which two balls are drawn. If the two balls drawn are of the same color, Andrew pays Bob 10\$, if the two balls are of two different colors, Bob plays Andrew x\$. How much is x if the game is fair?
Problem #4C$^{\bullet\bullet\bullet}$: Numbers $1, ..., 5$ are randomly distributed among the five players A, B, C, D and E, who play the following game. The first match is between A and B, the one who has the greater number proceeds and plays the second match with C, then the winner of the second match plays against D, and so on. Let X denote the number of matches won by A. Determine the probability mass function of the random variable X.
Problem #4D$^{\bullet\bullet\bullet}$: Let $S = \{1, 2, \ldots, n\}$, and suppose that A and B are, independently, equally likely to be any of the 2^n subsets (including the null set and S itself) of S.

(a) Show that $P\{A \subset B\} = \left(\frac{3}{4}\right)^n$.

(b) Show that $P\{A \cap B = \emptyset\} = \left(\frac{3}{4}\right)^n$.

Problem #4E$^{\bullet\bullet\bullet}$: Cities A, B, C, D are located (in this order) on the four corners of a square. Between them, we have the following roads: $A \leftrightarrow B$, $B \leftrightarrow C$, $C \leftrightarrow D$, $D \leftrightarrow A$, $B \leftrightarrow D$. One night each of these roads gets blocked by the snow independently with probability 1/2. Show that the next morning city C is accessible from city A with probability 1/2.
Bonus Problem #4F: Recall Pólya's urn model: initially, there are two balls in the urn, one blue ball and one red ball. At each step, a ball is drawn, its color is observed, and, in addition to the ball drawn, another ball of the same color is put into the urn. Determine the distribution of the number of red balls after n steps, that is, the probability that there are exactly k red balls in the urn ($k = 1, 2, \ldots, n, n + 1$).

Homework #5 (Due on March 31)

Chapter 4, Problem $4^{\bullet\bullet\bullet}$, $21^{\bullet\bullet}$, $26^{\bullet\bullet}$, $30^{\bullet\bullet}$, $37^{\bullet\bullet}$, $38^{\bullet\bullet\bullet}$,
Problem #5A$^{\bullet\bullet\bullet}$: Let N be a random variable that takes non-negative integer values. Show that

$$\mathbb{E}(N) = \sum_{i=1}^{\infty} \mathbb{P}(N \geq i).$$

(*Hint:* $\sum_{i=1}^{\infty} \mathbb{P}(N \geq i) = \sum_{i=1}^{\infty} \sum_{k=i}^{\infty} \mathbb{P}(N = k)$. Now interchange the order of the summation.)
Problem #5B$^{\bullet\bullet\bullet}$: A man has n keys, out of which there is just one that opens a specific lock. The man keeps trying the keys until he finds the right one. Compute the expected number of trials in both of the following cases: (a) he puts away the keys he has tried and turned out to be wrong, and at each trial he picks uniformly from the remaining ones (sampling without replacement); (b) at each trial, he picks one uniformly from all the n keys (sampling with replacement).

Homework #6 (Due on March 31)

Chapter 4, Problem $46^{\bullet\bullet\bullet}$, $50^{\bullet\bullet\bullet}$, $51^{\bullet\bullet}$, $56^{\bullet\bullet\bullet}$, $64^{\bullet\bullet\bullet}$,
Chapter 4, Theoretical Exercise $16^{\bullet\bullet\bullet}$, $25^{\bullet\bullet\bullet}$

Relevant pages of the book are available here:

`www.math.bme.hu/~pet/pro/HW5.pdf`

`www.math.bme.hu/~pet/pro/HW6.pdf`