3. gyakorlat

1/c) $B = \frac{1}{\sqrt{2}}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$ $\quad B^2 = \frac{1}{2}\cdot\begin{bmatrix}1&1\\1&-1\end{bmatrix}\cdot\begin{bmatrix}1&1\\1&-1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2&0\\0&2\end{bmatrix} = I$

$B^{2k+1} = B \qquad B^{2k} = I$

$C = \begin{bmatrix}1&1\\0&1\end{bmatrix} \qquad C^n := \begin{bmatrix}a_n&b_n\\c_n&d_n\end{bmatrix} \qquad C^{n+1} = \begin{bmatrix}a_n&b_n\\c_n&d_n\end{bmatrix}\cdot\begin{bmatrix}1&1\\0&1\end{bmatrix} = \begin{bmatrix}a_n&a_n+b_n\\c_n&c_n+d_n\end{bmatrix}$

$a_{n+1} = a_n = \ldots = a_1 = 1 \qquad b_{n+1} = a_n + b_n = a_n + \overbrace{a_{n-1} + \ldots + a_1 + b_1}^{b_n} = n+1$

$c_{n+1} = c_n = \ldots = c_1 = 0 \qquad d_{n+1} = c_n + d_n = c_n + \overbrace{c_{n-1} + \ldots + c_1 + d_1}^{d_n} = 1$

$C^n = \begin{bmatrix}1&n\\0&1\end{bmatrix}$ (Bizonyíthattuk volna indukcióval is)

Áll: $\begin{bmatrix}1&1\\0&1\end{bmatrix}^n = \begin{bmatrix}1&n\\0&1\end{bmatrix}$

(A) $n=1$ $\quad \begin{bmatrix}1&1\\0&1\end{bmatrix} \stackrel{?}{=} \begin{bmatrix}1&1\\0&1\end{bmatrix}$ ✓

(B) Tfh: $\begin{bmatrix}1&1\\0&1\end{bmatrix}^n = \begin{bmatrix}1&n\\0&1\end{bmatrix}$

(C) $\begin{bmatrix}1&1\\0&1\end{bmatrix}^{n+1} = \begin{bmatrix}1&1\\0&1\end{bmatrix}^n\cdot\begin{bmatrix}1&1\\0&1\end{bmatrix} \stackrel{(B)}{=} \begin{bmatrix}1&n\\0&1\end{bmatrix}\cdot\begin{bmatrix}1&1\\0&1\end{bmatrix} = \begin{bmatrix}1&n+1\\0&1\end{bmatrix}$ ✓

3/b) $\left[\begin{array}{ccc|c}1&2&a&0\\a&1&b&0\\1&2&-1&0\end{array}\right] \sim \begin{bmatrix}1&2&a\\0&1-2a&b-a^2\\0&0&-1-a\end{bmatrix}$

I $a=-1$ $\quad r(A)=r(A|b)=2 < 3$ $\quad \infty$ m.o.

II $a=1/2$ $\quad r(A)=r(A|b)=2$ $\quad \infty$ m.o.

$a \neq -1, 1/2 \quad x_1 = x_2 = x_3 = 0$

I $a=-1$

$x_1 + 2x_2 + x_3 = 0$

$3x_2 + b-1\, x_3 = 0$

$b=1 \Rightarrow x_2 = 0$ és $x_1 = x_3$

ált. m.o. $\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}\cdot x_3$

$b \neq 1 \quad x_2 = \frac{1-b}{3}x_3 \quad x_1 = x_3 - 2\frac{1-b}{3}x_3 = x_3\cdot\frac{1+2b}{3}$

ált. m.o. $\underline{x} = \begin{pmatrix}\frac{1+2b}{3}\\\frac{1-b}{3}\\1\end{pmatrix}\cdot x_3$

II $a = 1/2$

$x_1 + 2x_2 + \frac{1}{2}x_3 = 0$

$(-1-a)\cdot x_3 = 0$

$x_3 = 0 \qquad x_1 = -2x_2$

$\underline{x} = \begin{bmatrix}-2\\1\\0\end{bmatrix}\cdot x_2$