4. gyakorlat

1/a) $\underline{v}=(1,0,0)^T$ ir.-ú egyenesre vetítés mx-a $\dfrac{\underline{v}\cdot\underline{v}^T}{\underline{v}^T\underline{v}} = \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$

b) $\dfrac{\underline{v}\cdot\underline{v}^T}{\underline{v}^T\cdot\underline{v}}$

c) $\underline{n}$ nv.-ú síkra vetítés mx-a $(\mathbb{R}^3\to\mathbb{R}^3)$ $I - \dfrac{\underline{n}\cdot\underline{n}^T}{\underline{n}^T\cdot\underline{n}}$

d) $\underline{n}=(3,4,-12)^T$ nv.-ú síkra tükrözés mx-a $I-2\dfrac{\underline{n}\cdot\underline{n}^T}{\underline{n}^T\cdot\underline{n}} = \dfrac{1}{169}\cdot\begin{bmatrix}141 & \overset{-24}{\cancel{+24}} & +72\\ -24 & 137 & 96\\ 72 & 96 & -119\end{bmatrix}$

e) $\underline{v}=(6,-7,6)$ i.v.-ú egyenesre tükrözés $\cancel{+}-I+\dfrac{2\underline{v}\,\underline{v}^T}{\underline{v}^T\cdot\underline{v}} = \dfrac{1}{121}\cdot\begin{bmatrix}-49 & -84 & \cancel{+}72\\ -84 & -23 & -84\\ \cancel{+}72 & -84 & -49\end{bmatrix}$

f) $D: P_n \to P_n$ $\quad D(p(x)) = x\cdot p'(x) + 2p''(x) + 3p(x)$

$\varepsilon_k = x^k \quad (k=0\dots n)$

$\varepsilon_k \to (k+3)\cdot\varepsilon_k + 2k\cdot(k-1)\cdot\varepsilon_{k-2}$

$D(x^k) = k\cdot x^k + 2k(k-1)\cdot x^{k-2} + 3x^k$

$$\begin{array}{c|c}
 & \begin{matrix}0 & 1 & 2 & \dots & k & \dots & n\end{matrix}\\ \hline
\begin{matrix}0\\1\\2\\ \vdots\\k\\ \vdots\\n\end{matrix} & \begin{bmatrix}3 & 0 & 4 & & 0 & & 0\\ 0 & 4 & 0 & & & & \\ 0 & 0 & 5 & & 0 & & \vdots\\ & & & \ddots & 2k(k-1) & & \\ 0 & 0 & & & k+3 & & \vdots\\ & & & & & \ddots & 2n(n-1)\\ 0 & 0 & \dots & & & & n+3\end{bmatrix}
\end{array}$$

h) $A:\begin{pmatrix}r_1\\r_2\\r_3\end{pmatrix} \longrightarrow \begin{vmatrix}i & j & k\\ b_1 & b_2 & b_3\\ r_1 & r_2 & r_3\end{vmatrix} = \begin{pmatrix}b_2r_3 - b_3\cdot r_2\\ -b_1r_3 + b_3r_1\\ b_1r_2 - b_2r_1\end{pmatrix}$ $\quad A = \begin{bmatrix}0 & -b_3 & \cancel{+}b_2\\ b_3 & 0 & -b_1\\ -b_2 & b_1 & 0\end{bmatrix}$

2 b) $\begin{bmatrix}1 & 0 & 0\\ 0 & \sqrt3/2 & -1/2\\ 0 & 1/2 & \sqrt3/2\end{bmatrix} \begin{bmatrix}\sqrt3/2 & 0 & -1/2\\ 0 & 1 & 0\\ 1/2 & 0 & \sqrt3/2\end{bmatrix} \begin{bmatrix}\sqrt3/2 & -1/2 & 0\\ 1/2 & \sqrt3/2 & 0\\ 0 & 0 & 1\end{bmatrix} \cdot \begin{bmatrix}1\\1\\0\end{bmatrix}$

x tengely körül $\qquad$ y körül $\qquad$ z körül

$\begin{bmatrix}3/4 & -\sqrt3/4 & -1/2\\ \sqrt3/4 & -1/4 & +\sqrt3/2\\ & 1/2 & \end{bmatrix} \begin{bmatrix}\sqrt3/2 - 1/2\\ 1\\ 1/2\end{bmatrix}$

r képe: $\begin{bmatrix}\cancel{3/4}\,1/4 - \sqrt3/4\\ 9/8 - \sqrt3/8 - 1/4\\ \dfrac{3\sqrt3}{8} - \dfrac{1}{8} + \dfrac{\sqrt3}{4}\end{bmatrix} = \begin{bmatrix}1/4 - \sqrt3/4\\ 7/8 - \sqrt3/8\\ \sqrt3/8 - 1/8\end{bmatrix}$ $\leftarrow$ visszafelé írjuk

3 b)

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 3 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \overset{S_2-2S_1,\ S_3-3S_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -4 & -4 & -2 & 1 & 0 \\ 0 & -4 & -8 & -3 & 0 & 1 \end{array}\right] \overset{S_3-S_2,\ S_2/(-4)}{\sim} \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & +1/2 & -1/4 & 0 \\ 0 & 0 & -4 & -1 & -1 & 1 \end{array}\right] \sim$$

$$\overset{S_3/(-4),\ S_1-S_2,\ S_2-S_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & +1/2 & 0 \\ 0 & 1 & 0 & 1/4 & -1/2 & 1/4 \\ 0 & 0 & 1 & +1/4 & +1/4 & -1/4 \end{array}\right] \sim \left[\begin{array}{c|ccc} & -1/4 & 1/4 & 1/4 \\ I & 1/4 & -1/2 & 1/4 \\ & 1/4 & +1/4 & -1/4 \end{array}\right] \quad A^{-1} = \begin{bmatrix} -1/4 & 1/4 & 1/4 \\ 1/4 & -1/2 & 1/4 \\ 1/4 & 1/4 & -1/4 \end{bmatrix}$$