6. gyakorlat

① $\left[\begin{array}{ccc|c} 2a+1 & -a & a+1 & a-1 \\ a-2 & a-1 & a-2 & a \\ 2a-1 & a-1 & 2a-1 & a \end{array}\right] \sim$ ha $a \neq 2$ $\left[\begin{array}{ccc|c} a-2 & a-1 & & \\ 0 & -a-\frac{2a+1}{a-2}\cdot(a-1) & & \\ 0 & (a-1)(1-\ldots) & & \end{array}\right]$ $\begin{array}{c|c} a-2 & a \\ a+1-2a-1 & a-1-\frac{2a+1}{a-2}\cdot a \\ 0 & a\cdot\left(1-\frac{2a-1}{a-2}\right) \end{array}$

$s_1 - \frac{2a+1}{a-2} \cdot s_2 \qquad \frac{2a-1}{a-2}$

$s_3 - \frac{2a-1}{a-2} \cdot s_2$

$\sim \left[\begin{array}{ccc|c} a-2 & a-1 & a-2 & a \\ 0 & \frac{-3a^2-3a+1}{a-2} & -a & \frac{-a^2-4a+2}{a-2} \\ 0 & (a-1)\cdot\frac{-a-1}{a-2} & 0 & a\frac{-a-1}{a-2} \end{array}\right]$

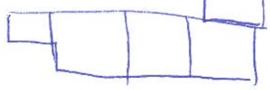
$a \neq 2 \Rightarrow r(A) = 2$ a.cs.a. 2. vagy 3. sor $\emptyset$.

a.cs.a. (~~...~~)

(most) 3. sor $\emptyset$ a.cs.a.

$a = 1,\ a = -1$

$a = 1 \Rightarrow$  $\emptyset$ mo.

$a = -1 \Rightarrow$  ∞ mo.

$a = 2 \quad \left[\begin{array}{ccc|c} 5 & -2 & 3 & 1 \\ 0 & 1 & 0 & 1 \\ 5 & 1 & 3 & 1 \end{array}\right] \Rightarrow x_2 = 0$

$x_1 + x_3 = 1/3$

$5x_1 + 3x_3 = 1 \quad 5\cdot(1/3 - x_3) + 3x_3 = 1$

$x_3 = \ldots$ 1 mo

$a \neq 1, -1 \Rightarrow 1$ mo.

④ $A = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 1 & 0 \\ -5 & 3 & 2 \end{bmatrix} \qquad \det(A - \lambda I) = \begin{vmatrix} -\lambda & 3 & 1 \\ 1 & 1-\lambda & 0 \\ -5 & 3 & 2-\lambda \end{vmatrix} =$

$1 \cdot \begin{vmatrix} 1 & 1-\lambda \\ -5 & 3 \end{vmatrix} + (2-\lambda) \cdot \begin{vmatrix} -\lambda & 3 \\ 1 & 1-\lambda \end{vmatrix} = 3 + 5 - 5\lambda + (2-\lambda)(\lambda^2 - \lambda - 3) =$

$-\lambda^3 + \lambda^2 + 3\lambda + 2\lambda^2 - 2\lambda - 6$

$= -\lambda^3 + 3\lambda^2 - 4\lambda + 2$

$= (\lambda - 1)\cdot(-\lambda^2 + 2\lambda - 2) \Leftarrow$

$\lambda = 1$ mo: $(-\lambda^3 + 3\lambda^2 - 4\lambda + 2) : (\lambda - 1) = -\lambda^2 + 2\lambda - 2$

$\ominus\ -\lambda^3 + \lambda^2$

$2\lambda^2 - 4\lambda + 2$

$\ominus\ 2\lambda^2 - 2\lambda$

$-2\lambda + 2$

$\lambda^2 - 2\lambda + 2 = 0$

$\lambda_{1,2} = \frac{+2 \pm \sqrt{4-8}}{2} = 1 \pm i \qquad 2i$

$\lambda = 1$-hez sv.:

$\begin{bmatrix} -1 & 3 & 1 \\ 1 & 0 & 0 \\ -5 & 3 & 1 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ ilyet keresünk!

$x = 0$ (∞ sok mo van egyet választhatunk szabadon!)

$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \leftarrow -3$ lesz

$\begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$

$\lambda = 1+i$-hez $\begin{bmatrix} -1-i & 3 & 1 \\ 1 & -i & 0 \\ -5 & 3 & 1-i \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

keressük így: $\begin{pmatrix} x \\ 1 \\ z \end{pmatrix} \xrightarrow{2.\ sor} \begin{pmatrix} i \\ 1 \\ z \end{pmatrix}$ 1. sor $-i+1+3+z=0$

$z=-4+i$

ell. 3. sor: $-5 \cdot i + 3 + (1-i)(-4+i) = -5i+3-4+4i+i+1=0$ ✓

$1+i$-hez sv: $\begin{pmatrix} i \\ 1 \\ -4+i \end{pmatrix}$ $1-i$-hez sv. (előző konjugáltja!

($A\underline{u} = \lambda \underline{u}$ konj: $\overline{A\underline{u}} = A\overline{\underline{u}} = \overline{\lambda}\,\overline{\underline{u}}$

$\rightarrow$ valós mx-nál

konj. sé-hez konj. svok tartoznak)

$1-i$-hez sv.: $\begin{pmatrix} -i \\ 1 \\ -4-i \end{pmatrix}$

c) $|A-\lambda I| = \begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 3-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} - 2\begin{vmatrix} 1 & 1 \\ 1 & 2-\lambda \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 3-\lambda \\ 1 & 2 \end{vmatrix}$

$= (2-\lambda)\cdot(6-5\lambda+\lambda^2-2) - 2(2-\lambda-1) + (2-3+\lambda) =$

$= 8-10\lambda+2\lambda^2-4\lambda+5\lambda^2-\lambda^3 - 2 + 2\lambda - 1 + \lambda =$

$= -\lambda^3 + 7\lambda^2 - 11\lambda + 5$ $\lambda = 1$ az mo.! $(-\lambda^3+7\lambda^2-11\lambda+5)/(\lambda-1) = (-\lambda^2+6\lambda-5)$

$\ominus\ -\lambda^3+\lambda^2$

$6\lambda^2-11\lambda+5$

$\ominus\ 6\lambda^2-6\lambda$

$-5\lambda+5$

$\ominus\ -5\lambda+5$

0

$\|$

$-(\lambda^2+6\lambda-5)\cdot(\lambda-1) = (\lambda-1)(\lambda-1)\cdot(\lambda-5)$

$\lambda^2-6\lambda+5=0$

$\lambda_{1,2} = \dfrac{+6 \pm \sqrt{36-20}}{2}$ $\begin{matrix} 5 \\ 1 \end{matrix}$

$\lambda = 1$-hez sv.: $\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

1 rangú $\Rightarrow$ van 2 lin ftlen !!! pl: $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

$\lambda = 5$-höz $\begin{pmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ pl: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ (és csak 1 van lin. ftlen.-ig)

5/b)

$$\begin{vmatrix} 5-\lambda & -4 & -2 \\ -4 & 5-\lambda & -2 \\ -2 & -2 & 8-\lambda \end{vmatrix} = -\lambda\cdot(\lambda-9)^2$$

$\lambda=0$-hoz: $\begin{bmatrix} 5 & -4 & -2 \\ -4 & 5 & -2 \\ -2 & -2 & 8 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

$\lambda=9$-hez: $\begin{bmatrix} -4 & -4 & -2 \\ -4 & -4 & -2 \\ -2 & -2 & -1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

↳ rang 1 $\Rightarrow$ 2 l.f. $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$

B bázisban: $\left(B = \left\{\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix}\right\}\right)$ $A_B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

← majdnem ONB!

$$A = T_{X\leftarrow B}\cdot A_B\cdot T_{B\leftarrow X} = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 0 & -4 \end{bmatrix}\cdot\begin{bmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 5 \end{bmatrix}\cdot\begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 0 & -4 \end{bmatrix}^{-1}$$

$$e^A = \begin{bmatrix} 2 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & -4 \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^9 & 0 \\ 0 & 0 & e^9 \end{bmatrix}\cdot\begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 0 & -4 \end{bmatrix}^{-1} = \dots$$

6) $\det A\neq 0 \iff 0$ nem sé.

Most A sé-ei $1, 2, 1/2$ $\qquad A\cdot\underline{v} = \lambda\underline{v} \iff \underline{v} = \lambda\cdot A^{-1}\cdot\underline{v}$

$A^{-1}\cdot\underline{v} = \frac{1}{\lambda}\cdot\underline{v}$

(Ha $\det A\neq 0 \Rightarrow$ A sé λ és hozzátartozó $\underline{v}$ sv $\Rightarrow$ A^{-1}-nek $\frac{1}{\lambda}$ sé $\underline{v}$ sv-val.

$\Rightarrow A^{-1}$ sé-ei $1, 2, 1/2$ Elég kiszámolni A-nak 2-höz tartozó sv-t!

$$\begin{vmatrix} -1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -3/2 \end{vmatrix}\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \\ 1 \\ 0 \end{pmatrix} \overset{1.\text{ sv}}{\Longrightarrow} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

egységhosszú A^{-1}-ben $1/2$-hez tartozó sv: $\frac{1}{\sqrt{5}}\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

9) $\lambda=2$ sv: $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\quad\lambda=3$-hoz: $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$-re $\perp$: $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow$

$B = \left\{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\}$ $\quad T_{X\leftarrow B} = \begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$$M = \underbrace{\begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{T_{X\leftarrow B}}\cdot\underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}}_{M_B}\cdot\begin{bmatrix} 0 & 1 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}^{-1} = \dots$$