9. Gyakorlat

① Áll: $\sum_{n=n_0}^{\infty} a_n$ K $\Leftarrow$ $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$
$\sum_{n=n_0}^{\infty} a_n$ D $\Leftarrow$ $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ } GYÖK kritérium

Áll: $\sum_{n=n_0}^{\infty} a_n$ K $\Leftarrow$ $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$
$\sum_{n=n_0}^{\infty} a_n$ D $\Leftarrow$ $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} > 1$ } HÁNYADOS kritérium

a) $\lim_{n\to\infty} \sqrt[n]{\frac{1}{(\ln n)^n}} = \lim_{n\to\infty} \frac{1}{\ln n} = 0 < 1 \Rightarrow K$

b) $\lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \frac{(2+\frac{1}{n})^n}{(2+\frac{1}{n+1})^{n+1}} = \lim_{n\to\infty} \frac{(n+1)}{2+\frac{1}{n+1}} \cdot \left(\frac{2+1/n}{2+1/n+1}\right)^n = \lim_{n\to\infty} \frac{(n+1)^2}{2n+3} \cdot \frac{\overbrace{\left(1+\frac{1}{2n}\right)^n}^{\to e^{1/2}}}{\underbrace{\left(1+\frac{1}{2(n+1)}\right)^n}_{\to e^{1/2}}} = \infty$

$\to D$

② c) $\sum_{n=2}^{\infty} \frac{1}{n \ln^p n}$ $\quad a_n = \frac{1}{n \ln^p n} \geq 0$ ✓ mon. csökk ?

$f(x) = \frac{1}{x \cdot \ln^p(x)}$ mon. csökk ?

$f'(x) = -(x \ln^p(x))^{-2} \cdot \left(\ln^p(x) + \cancel{x}\, p \ln^{(p-1)}(x) \cdot \frac{1}{\cancel{x}}\right) < 0 \Rightarrow$ mon. csökk ✓

$x \geq 2 \Rightarrow$ $\overset{\vee}{0}$ $\quad \ln(x) > 0$ $\quad \overset{\vee}{0}$

$f(n) = a_n \Rightarrow$ mon. csökk ✓

$$\int_2^{\infty} \frac{1}{x \ln^p x}\, dx = \int_2^{\infty} \frac{1}{x} \cdot (\ln x)^{-p} dx = \left[\frac{\ln(x)^{-p+1}}{-p+1}\right]_2^{\infty} =$$

$$= \lim_{x\to\infty} \frac{\ln(x)^{-p+1}}{-p+1} - \frac{\ln(2)^{-p+1}}{-p+1}$$

K, ha $p \geq 1$, akkor $= 0$
$-p+1 \leq 0$

Ha $\overset{0}{\hat{}}\, p < 1$ $\quad \sum_{n=2}^{\infty} \frac{1}{n \ln^p(n)}$

Nehéz

Ha $p < 0$ $\quad \lim_{n\to\infty} \frac{\ln^{-p}(n)}{n} = \infty \Rightarrow D$

Ha $p = 0$ $\quad \sum_{n=2}^{\infty} \frac{1}{n}$ D

③ c)

$$|S-S_k| = \left|\sum_{n=k+1}^{\infty} \frac{(-2)^n}{2^n+10^n}\right| < \left|\sum_{n=k+1}^{\infty} \frac{(-2)^n}{10^n}\right| = \frac{1}{5^{k+1}} \cdot \frac{1}{1+1/5}$$

$$\frac{1}{5^{k+1}} \cdot \frac{5}{6} \le 10^{-3} \iff 5^k \ge 1000/6$$

$$\underline{k \ge 4}$$

$\underline{S_4 \text{ jó!}}$

$\underline{\text{Leibniz kont}}$ $\sum_{n=1}^{\infty} \frac{(-2)^n}{2^n+10^n}$: alternál ✓

$\frac{2^n}{2^n+10^n} \to 0$ ✓

$$\frac{2^{n+1}}{2^{n+1}+10^{n+1}} \overset{?}{\le} \frac{2^n}{2^n+10^n} \iff 2^{n+1}+2\cdot 10^n \overset{?}{\le} 2^{n+1}+10^{n+1} \;✓ \quad \text{mon. csökk.}$$

$$\Rightarrow |S-S_k| \le c_{k+1} = \frac{2^{k+1}}{2^{k+1}+10^{k+1}} \le 10^{-3}$$

$k \ge 4$ az jó.

⑤ c)

$$0 \le \sum_{n=1}^{\infty} \frac{1}{2n^2-3n+8} \underset{\substack{\text{m.m.}\\ n\text{-re}}}{\le} \sum_{n=1}^{\infty} \frac{1}{2n^2-n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} \quad K$$

$3n < n^2$

$\Rightarrow$ a sor A.K.