1/ $\mathbb{P}(\overline{\bar{A} \cap B} \cup C) \overset{\uparrow}{=} \mathbb{P}(A \cup \bar{B} \cup C) \overset{\uparrow}{=} \mathbb{P}(A)+$

De Morgan   Poincaré formula

$\mathbb{P}(\bar{B}) + \mathbb{P}(C) - \mathbb{P}(A \cap \bar{B}) - \mathbb{P}(A \cap C) - \mathbb{P}(\bar{B} \cap C)$

$+ \mathbb{P}(A \cap \bar{B} \cap C) = \frac{1}{3} + \frac{2}{3} + \frac{1}{3} - \underbrace{\frac{1}{3} \cdot \frac{2}{3}}_{\substack{\mathbb{P}(A \cap \bar{B}) \\ = \mathbb{P}(A) \cdot \mathbb{P}(\bar{B})}} - \underbrace{0}_{\mathbb{P}(A \cap C) = 0}$

$- \underbrace{\frac{1}{3}}_{} + \underbrace{0}_{\mathbb{P}(A \cap \bar{B} \cap C) = 0} = \frac{7}{9}$

$\mathbb{P}(B \cap C) = 0$

$\Rightarrow C \subseteq \bar{B} \Rightarrow$

$\mathbb{P}(\bar{B} \cap C) = \mathbb{P}(C) = \frac{1}{3}$

2, AH, MH, FH

$\mathbb{P}(AH) = 0{,}15$

$\mathbb{P}(MH) = 0{,}3$

$\mathbb{P}(FH) = 0{,}2$

$\mathbb{P}(AH \cap MH) = \mathbb{P}(AH) \cdot \mathbb{P}(MH)$

mert minden párra.

De $\mathbb{P}(AH \cap MH \cap FH) = 0{,}02$.

$\mathbb{P}(\overline{AH} \cap \overline{MH} \cap \overline{FH}) = ?$

$\mathbb{P}(\overline{AH} \cap \overline{MH} \cap \overline{FH}) \overset{\uparrow}{=} \mathbb{P}(\overline{AH \cup MH \cup FH}) =$

~~Poincaré~~

De Morgan

$= 1 - \mathbb{P}(AH \cup MH \cup FH) \overset{\text{Poincaré}}{=} 1 - \mathbb{P}(AH) - \mathbb{P}(MH) - \mathbb{P}(FH) +$

$+ \mathbb{P}(AH \cap MH) + \mathbb{P}(AH \cap FH) + \mathbb{P}(MH \cap FH) -$

$- \mathbb{P}(AH \cap MH \cap FH) = 1 - 0{,}15 - 0{,}3 - 0{,}2 + 0{,}15 \cdot 0{,}3$

$+ 0{,}15 \cdot 0{,}2 + 0{,}2 \cdot 0{,}3 - 0{,}02 = \underline{\underline{0{,}465}}$

3) 5Z, 7K és 3Z, 8K

$\mathbb{P}(K_E) = ?$ $\quad \mathbb{P}(K_M) = ?$

Teljes vsz. tétele

$\mathbb{P}(K_E) = \mathbb{P}(K_E \mid E \xrightarrow{2\text{ kék}} M, M \xrightarrow{1\text{ kék}} E) \cdot \mathbb{P}(E \xrightarrow{2\text{ kék}} M, M \xrightarrow{1\text{ kék}} E) +$

$+ \dots + \mathbb{P}(K_E \mid E \xrightarrow{2\text{ zöld}} M, M \xrightarrow{1\text{ zöld}} E) \cdot \mathbb{P}(E \xrightarrow{2\text{ zöld}} M, M \xrightarrow{1\text{ zöld}} E)$

$= \frac{6}{11} \cdot \frac{\binom{7}{2}}{\binom{12}{2}} \cdot \cancel{\frac{\binom{10}{2}}{\binom{13}{2}}} \frac{10}{13} + \dots \approx \underline{0{,}594}$

1.: 5Z 7K

$\downarrow E \xrightarrow{2\text{ kék}} M$

2.: 5Z 5K

$\downarrow M \xrightarrow{1\text{ kék}} E$

3.: 5Z 6K

$\mathbb{P}(E \xrightarrow{2\text{ kék}} M) = \frac{\binom{7}{2}}{\binom{12}{2}}$

$\mathbb{P}(M \xrightarrow{1\text{ kék}} E \mid E \xrightarrow{2\text{ kék}} M) =$

$= \cancel{\frac{\binom{10}{2}}{\binom{13}{2}}} \frac{10}{13}$

4/ $\mathbb{P}(A)=0,75$

$\mathbb{P}(B)=0,15$

$\mathbb{P}(C)=0,1$

$\mathbb{P}(5|A)=0,4$

$\mathbb{P}(5|B)=0,7$

$\mathbb{P}(5|C)=0,6$

$$\mathbb{P}(A|5)\underset{\substack{\uparrow \\ \text{Bayes-} \\ \text{tétel}}}{=}\frac{\mathbb{P}(5|A)\cdot\mathbb{P}(A)}{\mathbb{P}(5|A)\cdot\mathbb{P}(A)+\mathbb{P}(5|B)\cdot\mathbb{P}(B)+\mathbb{P}(5|C)\cdot\mathbb{P}(C)}=\underline{\underline{0,6452}}$$

$\mathbb{P}(B|5)=\underline{\underline{0,2258}}$

$\mathbb{P}(C|5)=\underline{\underline{0,1290}}$

5/

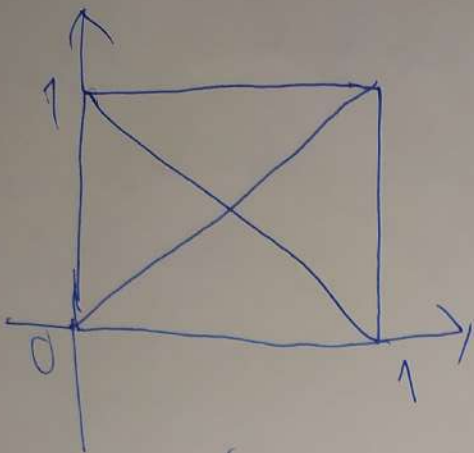
B dobása

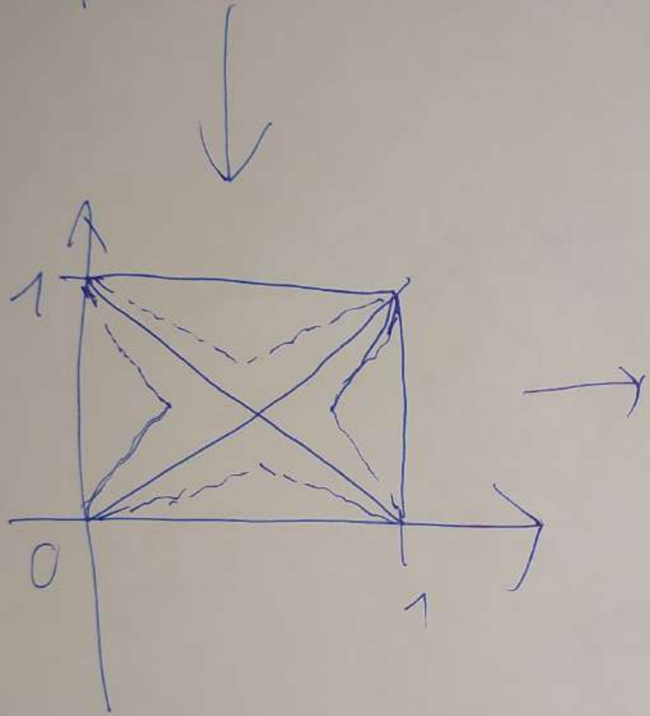
A nyereménye	1	2	3	4	5	6
1	0	-9	-12	-15	-18	-21
2	9	0	0	-18	-21	-24
3	12	0	0	0	0	-27
4	15	18	0	0	0	0
5	18	21	0	0	0	0
6	21	24	27	0	0	0

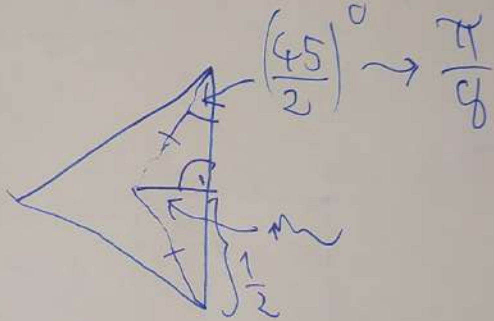
A dobása

$$\mathbb{E}(\text{A nyereménye})=\sum_{i=\pm 9,\pm 12,\ldots,\pm 27} i\cdot\mathbb{P}(\text{A nyereménye}=i)=\underline{\underline{0}}$$

6/







$$\Rightarrow \operatorname{tg}\frac{\pi}{8} = \frac{m}{\frac{1}{2}}$$

$$\Rightarrow m \approx 0{,}2071$$

$$\Rightarrow r = \frac{lT}{\ddot{o}T} = \frac{0{,}4142}{1}$$

7/ $f(X) = \alpha \cdot x^4$, ha $x \in (2,3)$, egyébként 0.

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_2^3 \alpha \cdot x^4\,dx = \alpha \cdot \frac{1}{5}\left[x^5\right]_2^3 =$$

$$= 1 \Rightarrow \cancel{x}\ \alpha = \frac{5}{211}$$

$$\mathbb{E}[X] = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_2^3 \frac{5}{211} \cdot x^5\,dx = \underline{\underline{2{,}626}}$$

$$F_X(t) = \int_{-\infty}^{t} f(x)\,dx \overset{t \in (2,3]}{=} \int_2^t \frac{5}{211} x^4\,dx = \frac{1}{211}\left(\cancel{x}\,t^5 - 2^5\right)$$

$$F_X(t) = \begin{cases} 0, \text{ ha } t \le 2 \\ \frac{1}{211}(t^5 - 2^5), \text{ ha } 2 < t \le 3 \\ 1, \text{ ha } 3 < t. \end{cases}$$

$$\left(\mathbb{P}(X > 12) = 0\right) \quad 1 - \mathbb{P}(X \le 12) = 1 - 1 = \underline{\underline{0}}$$

8/ $$\int_{-\infty}^{\infty} f_Y(t)\,dt = \int_{-5}^{-2} \frac{\alpha}{(1+t)^2}\,dt = \left[\frac{-\alpha}{(1+t)}\right]_{-5}^{-2} =$$

$$= \frac{-\alpha}{(1-2)} - \frac{-\alpha}{(1-5)} =$$

$$= 1 \Rightarrow \alpha = \frac{4}{3}$$

$$\mathbb{P}(-4 < Y < -3) = \int_{-4}^{-3} f_Y(t)\,dt =$$

$$= \int_{-4}^{-3} \frac{4}{3(1+t)^2}\,dt = \frac{2}{9}$$

9) X_1: szükséges próbálkozások száma az első találathoz.

$X_1 \sim Geo(0{,}2)$

$\mathbb{E}[X_1] = \frac{1}{0{,}2} = 5$

X_2: szükséges próbálkozások száma az első után a másodikhoz.

$\mathbb{E}[X_2] = 5$, mivel a geometriai eloszlás örökifjú!

Y: szükséges dobások száma a második találathoz

$$\mathbb{E}[Y] = \mathbb{E}[X_1 + X_2] \underset{\uparrow}{=} \mathbb{E}[X_1] + \mathbb{E}[X_2] = 5+5 = \underline{10}$$

$\mathbb{E}$ lin

10) X: Egy adott óra alatt bekövetkező hívások száma

$X \sim Poi(\lambda)$

$\mathbb{P}(X=0) = e^{-\lambda} = 0{,}25 \Rightarrow \underbrace{\lambda = -\ln(0{,}25)}$

a) $\mathbb{E}[3X] = 3 \cdot \mathbb{E}[X] = 3 \cdot -\ln(0{,}25) \approx \underline{4{,}1589}$

b) Y: 8 órából hány órában érkezik be legfeljebb 1 hívás

$Y \sim Binom(8, p)$, ahol

$$p = \mathbb{P}(X \leq 1) = e^{-\lambda} + \frac{\lambda^1}{1!} e^{-\lambda} \approx \underline{0{,}5965}$$

$$\mathbb{P}(Y \geq 2) = 1 - \mathbb{P}(Y=1) - \mathbb{P}(Y=0) = \ldots \underline{0{,}991}$$