Calculus 1, Practise Course

3rd week

I. Exponential and logarithmic functions

1. Convert the following expressions to the indicated base.
   (a) 2^x using base e
   (b) $3^{\sin x}$ using base e
   (c) $\log_2(x^2+1)$ using base e
   (d) $\ln|x|$ using base 5
   (e) $a^{1/\log_{10} a}$ using base 10, for $a > 0$ and $a \neq 1$

2. A capacitor is a device that stores electrical charge. The charge on a capacitor accumulates according to the function

$$Q(t) = a\left(1 - e^{-t/c}\right).$$

   where t is the time measured in seconds, and a ad $c > 0$ are physical constants. The *steady-state*[1] *charge* is the value that $Q(t)$ approaches as t becomes large.
   (a) Sketch the graph of the charge function for $t \geq 0$, using $a = 1$ and $c = 10$.
   (b) Vary the value of a while holding c fixed. Describe the effect on the curve. How does the steady-state charge vary with a?
   (c) Vary the value of c while holding a fixed. Describe the effect on the curve. How does the steady-state charge vary with c?
   (d) Find a formula that gives the steady-state charge in terms of a and c.

3. Without using a graphing utility, sketch the graph of $y = 2^x$. Then in the same coordinate system, sketch the graphs of

[1] steady-state = egyensúlyi

(a) $y = 2^{-x}$

(b) $y = 2^{x-1}$

(c) $y = 2^x + 1$

(d) $y = 2^{2x}$

4. Without using a graphing utility, sketch the graph of $y = \log_2 x$. Then in the same coordinate system, sketch the graphs of

(a) $y = \log_2(x - 1)$

(b) $y = \log_2 x^2$

(c) $y = (\log_2 x)^2$

(d) $y = \log_2 x + 1$

II. Trigonometric functions and their inverses

1. Solve the following trigonometric equations.

(a) $\sin^2\theta = \frac{1}{4}$, $0 \le \theta < 2\pi$

(b) $\sin 3\theta = \frac{\sqrt{2}}{2}$, $0 \le \theta < 2\pi$

(c) $\cos 3x = \sin 3x$, $0 \le x < 2\pi$

(d) $\tan^2 2\theta = 1$, $0 \le \theta < 2\pi$

2. Without using a calculator, evaluate the following expressions or state that the quantity is undefined.

(a) $\sin^{-1} 1$

(b) $\cos^{-1}(-1)$

(c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

(d) $\sin^{-1}\frac{\sqrt{3}}{2}$

(e) $\cos^{-1} 2$

(f) $\cos^{-1}(\cos(7\pi/6))$

(g) $\cos(\cos^{-1}(-1))$

(h) $\tan^{-1}\sqrt{3}$

(i) $\tan^{-1}(\tan(\pi/4))$

(j) $\tan^{-1}(\tan(3\pi/4))$

3. Use a right-triangle sketch to complete the following exercises.

   (a) Suppose $\theta = \cos^{-1}(5/13)$. Find $\sin\theta$ and $\tan\theta$.

   (b) Suppose $\theta = \tan^{-1}(4/3)$. Find $\sin\theta$ and $\cos\theta$.

4. *[2] Draw a right triangle to simplify the given expressions. Assume $x > 0$.

   (a) $\cos(\sin^{-1} x)$

   (b) $\cos(\sin^{-1}(x/3))$

   (c) $\sin(\cos^{-1}(x/2))$

   (d) $\sin^{-1}(\cos\theta)$, $\theta \in [0, 2\pi]$

   (e) $\sin(2\cos^{-1} x)$

   (f) $\cos(2\sin^{-1} x)$

   (g) $\cos(\tan^{-1} x)$

   (h) $\tan(\cos^{-1} x)$

   (i) $\cos\left(\tan^{-1}\left(\frac{x}{\sqrt{9-x^2}}\right)\right)$

5. **[3] Prove the following identities.

   (a) $\cos^{-1} x + \cos^{-1}(-x) = \pi$

   (b) $\sin^{-1} x + \sin^{-1}(-x) = 0$

6. Identify the amplitude and period of the following functions.

   (a) $f(x) = 2\sin 2x$

   (b) $f(x) = 3\cos(x/3)$

   (c) $f(t) = 5\sin\left(\frac{1}{2}(t-3)\right)$

   (d) $f(x) = 3\cos(\pi x/24)$

   (e) $f(x) = \sin 2\pi x$

   (f) ** $f(x) = \sin^4 x + \cos^4 x$

   (g) * $f(x) = |\cos x|$

7. Sketch the graph of the following functions

   (a) $f(x) = 3\sin 2x$

[2] *: more challanging tasks

[3] **: problems for brave hearted

(b) $g(x) = 3\sin(2x - \pi/3) + 1$

(c) $h(x) = -2\cos(\pi x/24) + 2$

8. Design a sine function with the given properties.

   (a) It has a period of 12 with a minimum value of -4 at $t = 0$ and a maximum value 4 at $t = 6$.

   (b) It has a period of 24 with a minimum value of 10 at $t = 3$ and a maximum value 16 at $t = 15$.