Calculus 1, Practise Course

6th week

I. Limits at infinity

1. Determine the following limits.

   (a) $\lim_{x\to\infty}\left(3+\frac{17}{x^3}\right)$

   (b) $\lim_{x\to\infty}\frac{3x^4-17x^3+12x^2+x}{12x^4-6x^3+8x+3}$

   (c) $\lim_{x\to-\infty}\frac{3x^2-16x}{7x-8}$

   (d) $\lim_{x\to-\infty}\frac{40x^4+x^2+5x}{\sqrt{64x^8+x^6}}$

   (e) $\lim_{t\to\infty}\frac{3t^3-5t^2+t}{\sqrt[3]{8t^9+16t^7+5t^2+6}}$

   (f) $\lim_{x\to-\infty}\frac{\sqrt{16x^2+x}}{3x+6}$

   (g) $\lim_{x\to\infty}\frac{6x^2}{4x^2+\sqrt{16x^4+x^2}}$

   (h) $\lim_{x\to\infty}(x^2-\sqrt{x^4+3x^2})$

   (i) $\lim_{x\to-\infty}(x+\sqrt{x^2-5x})$

   (j) $\lim_{x\to-\infty}(e^x\cos x+3)$

   (k) $\lim_{x\to\infty}\frac{\sin 6x}{e^{2x}}$

   (l) $\lim_{x\to\infty}(\sqrt{x^2+1}-\sqrt{x^2-1})$

   (m) $\lim_{x\to\infty}\left(\frac{1-2x}{\sqrt[3]{1+8x^3}}+2^{-x}\right)$

   (n) * $\lim_{x\to\infty}(\sqrt[3]{1-x^3}+x)$

   (o) * $\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

   (p) $\lim_{x\to\infty}\frac{x^2(1+\sin^2 x)}{(x+\sin x)^2}$

2. ** The function f is defined by the following limit

$$f(x) = \lim_{t\to\infty} \frac{x^{2t} - 1}{x^{2t} + 1}.$$

Investigate this function and graph it.

II. Asymptotes (horizontal, vertical and slant)

1. Determine $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following functions. Then give the horizontal asymptotes of f if they exist.

   (a) $f(x) = \frac{6x^2-9x+8}{3x^2+2}$

   (b) $f(x) = \frac{3x^3-7}{x^4+5x^2}$

   (c) $f(x) = \frac{6x^2+1}{\sqrt{4x^4+3x+1}}$

   (d) $f(x) = \frac{1}{2x^4-\sqrt{4x^8-9x^4}}$

   (e) $f(x) = x - \sqrt{x^2 - 9x}$

   (f) $f(x) = 4x(3x - \sqrt{9x^2 + 1})$

2. Find the vertical asymptotes of the following functions. For each vertical asymptote $x = a$, analyze $\lim_{x\to a+} f(x)$ and $\lim_{x\to a-} f(x)$.

   (a) $f(x) = \frac{\sqrt{16x^4+64x^2}+x^2}{2x^2-4}$

   (b) $f(x) = \frac{x^2-9}{x(x-3)}$

   (c) $f(x) = \frac{x^4-1}{x^2-1}$

   (d) $f(x) = \frac{|1-x^2|}{x(x+1)}$

   (e) $f(x) = 3^{\frac{1}{x+2}}$

   (f) $f(x) = 3^{\frac{1}{|x-2|}}$

3. Find the slant (oblique) asymptotes of the following functions if they exist.

   (a) $f(x) = \frac{x^2-3}{x+6}$

   (b) $f(x) = \frac{x^2-2x+5}{3x-2}$

   (c) $f(x) = \frac{4x^3+4x^2+7x+4}{x^2+1}$

   (d) * $f(x) = \frac{x}{\sqrt{1-x^2}}$

(e) * $f(x) = \sqrt{x^2+3x-1}$

(f) * $f(x) = \frac{\sqrt{4x^4+1}}{|x|}$

III. Continuity

1. Determine whether the following functions are continous at the given x_0 point.

   (a)
$$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & \text{if } x \neq 1 \\ 3, & \text{if } x = 1 \end{cases}, \quad x_0 = 1.$$

   (b)
$$f(x) = \begin{cases} \frac{x^2-4x+3}{x-3}, & \text{if } x \neq 3 \\ 2, & \text{if } x = 3 \end{cases}, \quad x_0 = 3.$$

   (c)
$$f(x) = \begin{cases} \frac{x^2+x}{x+1}, & \text{if } x \neq -1 \\ 2, & \text{if } x = -1 \end{cases}, \quad x_0 = -1.$$

   (d) **
$$f(x) = \begin{cases} \frac{\sqrt{1+x+x^2}-1}{\sin 4x}, & \text{if } x \neq 0 \\ \frac{1}{8}, & \text{if } x = 0 \end{cases}, \quad x_0 = 0.$$

2. Determine the interval(s) on which the following functions are continous. At which finite endpoints of the intervals of continuity is the function continous from the left or from the right?

   (a) $f(x) = \frac{x^5+6x+17}{x^2-9}$

   (b) $g(x)\frac{3x^2-6x+7}{x^2+x+1}$

   (c) $f(x) = \sqrt{2x^2-16}$

   (d) $f(x) = (2x-3)^{2/3}$

   (e) $g(x) = \sqrt[3]{x^2-2x-3}$

   (f) $*h(x) = e^{\sqrt{x-1}}$

   (g) $*f(t) = e^{\frac{1}{\sqrt{t-1}}}$

   (h) $f(x) = \frac{e^x}{1-e^x}$

   (i) $f(x) = \frac{e^{2x}-1}{e^x-1}$

(j) $f(x) = \frac{1=\sin x}{\cos x}$

3. Classify the discontinuities (removable, jump or infinite discontinuities) in the following functions.

   (a) $f(x) = \frac{4}{x^2-2x+1}$

   (b) $f(x) = \frac{x^2-7x+10}{x-2}$

   (c) $f(x) = x + \frac{x+2}{|x+2|}$

   (d) $f(x) = \frac{2|x-1|}{x^2-x^3}$

   (e) $*f(x) = \frac{3(1-x^2)+|1-x^2|}{2(1-x^2)-|1-x^2|}$

   (f) $**f(x) = 3 + \frac{1}{1+3^{\frac{1}{1-x}}}$

   (g) $f(x) = \frac{x^2+2x-3}{x^2+5x+6}$

   (h) $f(x) = \frac{x^2-9}{x^2(x-3)^2}$

   (i) $*f(x) = 3^{\frac{1}{x+1}}$

4. Give the value of parameter(s) (a, b) to make the functions continous.

   (a)

$$f(x) = \begin{cases} ax^2 + 1 & \text{ha } x \geq 0, \\ -x & \text{ha } x < 0. \end{cases}$$

   (b)

$$f(x) = \begin{cases} (x-1)^3 & \text{ha } x \leq 0, \\ ax + b & \text{ha } 0 < x < 1, \\ \sqrt{x} & \text{ha } x \geq 1. \end{cases}$$

   (c)

$$f(x) = \begin{cases} x & \text{ha } |x| \leq 1, \\ x^2 + ax + b & \text{ha } |x| > 1. \end{cases}$$