

Calculus 1, Practise Course

11th week

I. Basic indefinite integrals

1. Verify the following indefinite integrals.

$$(a) \int \frac{\cos \sqrt{x}}{\sqrt{x}} \, dx = 2 \sin \sqrt{x} + C$$

$$(b) \int \frac{x}{\sqrt{x^2+1}} \, dx = \sqrt{x^2+1} + C$$

$$(c) \int x^2 \cos x^3 \, dx = \frac{\sin x^3}{3} + C$$

$$(d) \int \frac{\sin x}{\cos^2 x} \, dx = \frac{1}{\cos x} + C$$

2. Determine the following indefinite integrals.

$$(a) \int \frac{x^2-7x+8}{x^2} \, dx$$

$$(b) \int \frac{x^2}{x^2+1} \, dx$$

$$(c) \int \left(\sqrt{x} \sqrt[3]{x} \sqrt[4]{x} + \frac{1}{\sqrt{x}} \right) \, dx$$

$$(d) \int \frac{-2}{5x^2+5} \, dx$$

$$(e) \int \frac{2x+3}{x-2} \, dx$$

$$(f) \int \frac{1-e^x}{1+e^{x/2}} \, dx$$

$$(g) \int \frac{e^{2x}+1}{e^{3x}} \, dx$$

$$(h) \int \frac{t^2-e^{2t}}{t+e^t} \, dt$$

$$(i) \int \frac{1}{3x^2+6} \, dx$$

$$(j) \int \sqrt{3x-1} \, dx$$

$$(k) \int \frac{1}{\sqrt[3]{3x-2}} \, dx$$

$$(l) \int \frac{y^3-9y^2+20y}{y-4} \, dy$$

$$(m) \int \sqrt{x}(2x^6 - 4\sqrt[3]{x}) dx$$

$$(n) \int \frac{\cos 2x}{\cos x - \sin x} dx$$

3. Determine the following indefinite integrals with finding complete squares.

$$(a) \int \frac{dx}{x^2+2x+6}$$

$$(b) \int \frac{dx}{x^2-6x+1}$$

$$(c) \int \frac{dx}{2x^2-12x+23}$$

$$(d) \int \frac{dx}{\sqrt{4-9x^2}}$$

$$(e) \int \frac{dx}{\sqrt{x^2-4x+40}}$$

$$(f) \int \frac{dx}{\sqrt{x^2+6x}}$$

$$(g) \int \frac{dx}{\sqrt{5-x^2-4x}}$$

$$(h) \int \frac{dx}{x^2+4x+5}$$

$$(i) * \int \frac{dx}{4-x^2-4x}$$

4. $\int f'(x)f^\alpha(x) dx = \frac{f^{\alpha+1}(x)}{\alpha+1} + C$, ($\alpha \neq -1$) type

$$(a) \int \frac{2x-5}{\sqrt[3]{(x^2-5x+10)^7}} dx$$

$$(b) \int \sin^3 x \cos x dx$$

$$(c) \int \frac{\tan x}{\cos^2 x} dx$$

$$(d) \int \cos^5 x dx$$

$$(e) \int \sin^3 x dx$$

$$(f) \int \frac{\cos x}{\sin^2 x} dx$$

$$(g) \int \frac{\operatorname{ch} x}{\sqrt[3]{\operatorname{sh} x}} dx$$

$$(h) \int \frac{\sqrt{\ln^3 x}}{x} dx$$

$$(i) \int \frac{e^x}{e^{2x}+2e^x+1} dx$$

5. $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$ type

$$(a) \int \frac{5x^2}{5-4x^3} dx$$

$$(b) \int \tan x dx$$

$$(c) \int \frac{1}{x \ln x} dx$$

$$(d) \int \frac{9x^2}{4x^3+5} dx$$

$$(e) \int \frac{e^{3x}}{e^{3x}+6} dx$$

$$(f) \int \frac{1}{(1+x^2)\operatorname{arctg} x} dx$$

II. Integration by parts

1. Evaluate the following integrals.

$$(a) \int x^2 \sin 2x dx$$

$$(b) \int e^{2x} \cos 3x dx$$

$$(c) \int e^x \sin^2 x dx$$

$$(d) \int x^2 e^{-2x} dx$$

$$(e) \int x \operatorname{arctg} x dx$$

$$(f) \int \arcsin x dx$$

$$(g) \int x^9 \ln x dx$$

$$(h) \int \ln^3 x dx$$

$$(i) \int x \operatorname{sh} x dx$$

$$(j) \int \frac{\ln \cos x}{\cos^2 x} dx$$

$$(k) \int \frac{x^2 \operatorname{arctg} x}{1+x^2} dx$$

$$(l) \int \cos x \operatorname{sh} x dx$$

$$(m) * \int \frac{x e^x}{(x+1)^2} dx$$

$$(n) \int \cos(\ln x) dx$$

$$(o) * \int x \ln \left(1 + \frac{1}{x}\right) dx$$

$$(p) \int 3^x \cos x dx$$

$$(q) * \int \frac{x}{\sqrt{x+1}} dx$$

$$(r) * \int x \ln x^2 dx$$