Formula sheet: large deviation rate functions, exponential tilting

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Bernoulli distribution: If $X \sim \text{BER}(p)$ then

$$\widehat{I}(\lambda) = \ln\left(pe^{\lambda} + (1-p)\right), \quad I(x) = x\ln\left(\frac{x}{p}\right) + (1-x)\ln\left(\frac{1-x}{1-p}\right)$$

Binomial distribution: If $X \sim \text{BIN}(n,p)$ then

$$X^{(\lambda)} \sim \text{BIN}\left(n, \frac{pe^{\lambda}}{(1-p)+pe^{\lambda}}\right)$$

Poisson distribution: If $X \sim \text{POI}(\mu)$ then

$$\widehat{I}(\lambda) = \mu e^{\lambda} - \mu, \quad I(x) = x\ln\left(\frac{x}{\mu}\right) + \mu - x, \quad X^{(\lambda)} \sim \text{POI}(\mu e^{\lambda})$$

Exponential distribution: If $X \sim \text{EXP}(\mu)$ then

$$\widehat{I}(\lambda) = \ln\left(\frac{\mu}{\mu-\lambda}\right), \quad I(x) = \mu x - \ln(\mu x) - 1, \quad X^{(\lambda)} \sim \text{EXP}(\mu - \lambda)$$

Optimistic geometric distribution: If $X \sim \text{GEO}(p)$ and $q = 1-p$ then

$$\widehat{I}(\lambda) = \ln\left(\frac{p}{e^{-\lambda} - q}\right), \quad I(x) = (x-1)\ln\left(\frac{x-1}{q}\right) - x\ln(x) - \ln(p), \quad X^{(\lambda)} \sim \text{GEO}(1-qe^{\lambda})$$

Normal distribution: If $X \sim \mathcal{N}(m,\sigma^2)$ then

$$\widehat{I}(\lambda) = m\lambda + \frac{\sigma^2\lambda^2}{2}, \quad I(x) = \frac{(x-m)^2}{2\sigma^2}, \quad X^{(\lambda)} \sim \mathcal{N}(m+\lambda\sigma^2, \sigma^2)$$