MATH302, July 29: Some exercises on the normal distribution

Recall:
A r.v. X is normally distributed with parameters μ and σ if X has the following p.d.f.:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \qquad \text{for all} \qquad -\infty < x < \infty.$$

If X is normally distributed with parameters μ and σ, then we write $X \sim \mathcal{N}(\mu, \sigma)$. We have

$$\mathbf{E}(X) = \mu, \qquad \sqrt{\mathrm{Var}(X)} = \sigma.$$

Standard normal distribution:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \qquad X \sim \mathcal{N}(0, 1).$$

Standardization:

$$X \sim \mathcal{N}(\mu, \sigma), \quad Z := \frac{X - \mu}{\sigma} \quad \Longrightarrow \quad Z \sim \mathcal{N}(0, 1)$$

Table of the standard normal cumulative density function Φ contains the values of

$$\Phi(z) := \mathbf{P}(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, \mathrm{d}x \qquad \text{where } Z \sim \mathcal{N}(0,1), \ \ z \in \mathbb{R}.$$

Remember:

$$\Phi(-z) = 1 - \Phi(z), \quad \Phi(0) = \frac{1}{2}, \quad \mathbf{P}(Z \ge z) = 1 - \Phi(z).$$

Exercises:

1. The annual rainfall in Rain city in centimetres is distributed as a normal random variable with mean $\mu = 110$ cm, and standard deviation $\sigma = 10$ cm.
   (a) Find $\mathbf{P}$(annual rainfall exceeds 135 cm).
   (b) Find $\mathbf{P}$(annual rainfall is between 95 and 125 cm).
   (c) Find $\mathbf{P}$(it will take more than 10 years until the annual rainfall exceeds 135 cm).

2. The annual return of a stock is normally distributed with mean 10% and standard deviation 12%. If I buy 100 shares at \$60 each, what is the probability that after one year my net profit is at least \$750?

3. The scores of a test given to 100,000 students are normally distributed with mean 500 and standard deviation 100. What score will place a student in the top 10%?