

Math 302 HW assignment 1. Due Friday, July 11 at start of class

Note: Show all your work! In all solutions involving permutations, combinations, etc., be sure to briefly explain why they appear. Separate the solutions of different exercises with a line. Draw a frame around your final answer (if it is a number).

1. Five race horses enter a race (Seabiscuit, Red Rum, Desert Orchid, Black Caviar, Overdose). The horses are equally good, so all outcomes of the race are equally likely.
 - (a) (2 mark) How many possible outcomes are there in which Desert Orchid is first and Black Caviar is last?
 - (b) (2 mark) What is the probability that Red Rum is not the last one?
 - (c) (3 marks) How many possible outcomes are there in which Overdose beats Desert Orchid, who beats Red Rum?

2. I have 12 LEGO blocks: 3 green, 5 red, 4 blue. How many different LEGO towers can I build ...
 - (a) (2 marks) if I want the block on top to be red?
 - (b) (3 marks) if I want all the red blocks to be higher than all the blue blocks?
 - (c) (3 marks) if I don't want the three bottom blocks to be green?

3. Three families go to see a movie together. The Smiths have 4 family members, the Taylors have 5 and the Millers have 4. Each family has two parents and the rest are children (all distinguishable). One row in the cinema has 13 seats, so the three families together occupy a row. How many ways can they sit ...
 - (a) (3 marks) if each parent wants to sit next to their spouse?
 - (b) (3 marks) if each of the families want to sit together?
 - (c) (4 marks) if no two Millers are allowed to sit next to each other?

4. Google "list of poker hands wikipedia". What is the probability that a poker hand is...
 - (a) (3 marks) a "straight flush"?
 - (b) (3 marks) a "flush" (but not a straight flush)?
 - (c) (4 marks) a "two pairs" ?

Note: please write you answer using the notation used in class. You don't need to numerically evaluate the answers: the exact formula involving binomial coefficients is enough.

5. Use the binomial theorem to...
 - (a) (2 marks) find the closed form of $\sum_{j=0}^{50} \binom{50}{j} (-1)^j$.
 - (b) (2 marks) expand $(x + 2y)^{80}$ as a sum of monomials.

6. If 18 orphan cats are divided among 6 cat orphanages, how many divisions are possible ...
 - (a) (2 mark) if the cats are distinguishable and no further restriction is imposed?
 - (b) (3 marks) if the cats are distinguishable and each orphanage must receive 3 cats?
 - (c) (3 marks) if the cats are indistinguishable and each orphanage must receive at least one cat?

7. (3 marks) Verify and explain the identity $\binom{100}{30} \cdot \binom{70}{25} \cdot \binom{45}{5} = \binom{100}{30,25,5,40}$.