

**Math 302 HW assignment 3. Due Friday, July 25 at start of class**

*Note:* Write your student number on each page that you submit. Show all your work! Separate the solutions of different exercises with a line. Draw a frame around your final answer (if it is a number).

1. I throw three dice and denote the outcomes by  $X_1, X_2, X_3$ . Let us define the events

$$A = \{X_1 \neq X_2\}, \quad B = \{X_2 \neq X_3\}, \quad C = \{X_1 \neq X_3\}.$$

- (a) (3 marks) Calculate  $\mathbf{P}(A)$ ,  $\mathbf{P}(A|B)$  and  $\mathbf{P}(A|B \cap C)$ .  
 (b) (2 marks) Are  $B$  and  $C$  independent? Are  $A$ ,  $B$  and  $C$  completely independent? Why?
2. (2 marks) In a class there are four freshman boys, six freshman girls, six sophomore boys and a certain number of sophomore girls. When a student is selected at random, the following two events are independent:

$$A = \{ \text{the selected student is a boy} \} \quad \text{and} \quad B = \{ \text{the selected student is a freshman} \}.$$

What is the number of sophomore girls?

3. (4 marks) On a roulette wheel there are 18 black and 18 red numbers, plus the number 0 which is green. Before I start I have 100 dollars. My strategy is to keep betting 100 dollars on red again and again until I have either 0 or 1000 dollars. Denote by  $X$  my net profit (i.e., the difference between my wealth before starting and after finishing the game). What is  $\mathbf{E}(X)$ ?
4. Two equally good ping pong players (Alice and Bob) compete.
- (a) (2 marks) Assuming that the game goes on indefinitely, let us denote by  $X$  the number of rounds they that need to play until Bob wins 6 rounds. What is the distribution of  $X$ ?  
*Note:* It is a famous distribution: name it and specify its parameter(s).
- (b) (2 marks) What is  $\mathbf{E}(X)$ ? Why?
- (c) (3 marks) They compete to see who will be the first to win 11 games. The game gets interrupted when when Bob won 5 rounds and Alice won 8 rounds. What is the fair division of the price money (100 dollars) between Alice and Bob given this information?  
*Note:* Express your answer using the probability mass function of  $X$ .
5. In Dragonland the frequency of an  $n$ -headed dragon is

$$p_n = \binom{6}{n-1} \cdot 0.7^{n-1} \cdot 0.3^{7-n}, \quad n = 1, 2, \dots, 7.$$

Beheading a dragon is a dangerous business: one succeeds in cutting off each head independently from the others with 90% chance, and if he/she fails to cut it off, then the head eats him/her.

- (a) (2 mark) What is the average number of heads of a dragon from Dragonland?  
*Hint:* Use your knowledge about the expectation of famous random variables.
- (b) (2 marks) I come across a dragon, but I can't see from the fog how many heads it has. What is my chance of surviving this encounter?
- (c) (2 marks) After the battle I meet a friend who also just killed a dragon. What is the probability that it was 7-headed?
6. Alice and Bob play target shooting. With each shot, Alice hits the target with 15% chance, Bob hits it with 10% chance. Bob starts and they take turns. Whoever hits the target first wins.
- (a) (3 marks) What is the probability that Alice wins?
- (b) (3 marks) What is the expectation of the number of bullets shot throughout the game?