Stoch. Anal. HW assignment 1. Due Friday, September 15 at start of class

Note: Each of the 3 questions is worth 10 marks. Write your name and Neptun code on each piece of paper that you submit. Separate the solutions of different exercises with a horizontal line. Highlight the final answer. If you submit your homework electronically, pdf format is preferred.

1. A heavy smoker tries to quit smoking from time to time. After a year of smoking, on New Year's Eve he makes the resolution of quitting smoking with probability 2/3. If he makes this resolution, he will be abstinent all year next year. If he decides not to make this resolution, he will continue smoking next year. After a year of abstinence, with probability 3/4 he becomes so proud of himself that on New Year's Eve he allows himself to smoke for a year; otherwise he continues being abstinent for a year. He continues with the same pattern each year. Since he also drinks heavily on New Year's Eve, his memory span becomes very limited, so he makes his decision about smoking in the upcoming year solely based on his smoking habits in the current year.

   If he smokes in 2017, what is the probability that he will smoke in 2027? Find a simple exact formula.

   Hint: This is similar to the „volleyball" exercise solved in class.

2. We consider a simple random walk on the graph $\mathbb{Z}$: in each step, the walker tosses a fair coin and makes one step to the left if the coin comes up heads and makes one step to the right if the coin comes up tails. The walker starts from the origin.

   (a) What is the probability that the walker reaches site -100 before reaching site 200?
   (b) What is the expected number of visits to site 50 before exiting the interval $[-99, 199]$?

   Hint: Yes, you have to solve a system of linear equations with hundreds of variables and equations, but it's not that hard if you try.

3. Consider two dogs (a dachshund and a beagle) and three fleas. The fleas jump back and forth between the two dogs. From time to time, a uniformly chosen flea jumps from one dog to the other. Initially all of the fleas are on the dachshund. What is the expected total number of flea-jumps until the first time that all of the fleas are on the beagle?

   Hint: It is enough to keep track of the number of fleas on the beagle.