

## Stoch. Proc. HW assignment 4. Due Friday, October 6 at start of class

*Note:* Each of the 3 questions is worth 10 marks. Write your name and Neptun code on each piece of paper that you submit. Separate the solutions of different exercises with a horizontal line. Highlight the final answer. Paper format is preferred, but if you submit your homework electronically, pdf format is preferred.

1. Each day it rains with probability  $1/4$ , independently from what happens on other days. Moreover if it rains then it rains exactly at noon. An old gardener waters his garden in the afternoon if he sees that the garden has not been watered (by either rain or himself) in the last three days (i.e., today, yesterday and the day before yesterday). Roughly how many times does he water his garden per year?

*Hint:* Use the law of large numbers for Markov chains (see page 27-28 of the scanned lecture notes and the „official” solution of Homework 2.2).

2. There is a queue of university students standing in line in front of the Central Office of Education. Each second, a new student arrives and joins the queue with probability  $p$  and a student is served in the office and thus leaves the queue with probability  $q$ . The maximal length of the queue is 30. Any student arriving when there are already 30 students standing in line will be immediately sent away.
  - (a) If  $p = 0.01$  and  $q = 0.012$  and assuming that the queue is stationary, what is the probability that the length of the queue is at most 15?
  - (b) If  $p = 0.01$  and  $q = 0.008$  and assuming that the queue is stationary, what is the probability that the length of the queue is at most 15?
  - (c) If  $p = 0.01$  and  $q = 0.01$  and assuming that the queue is stationary, what is the probability that the length of the queue is at most 15?
  - (d) If  $p = 0.01$  and  $q = 0.012$  and the current queue length is 15, what is the probability that the queue becomes empty before it reaches its maximal length?
  - (e) If  $p = 0.01$  and  $q = 0.008$  and the current queue length is 15, what is the probability that the queue becomes empty before it reaches its maximal length?
  - (f) If  $p = 0.01$  and  $q = 0.01$  and the current queue length is 15, what is the probability that the queue becomes empty before it reaches its maximal length?

*Hint:* Since both  $p$  and  $q$  are very small numbers, it is OK ignore the possibility of two „events” (i.e., arrivals/departures) happening in the same second – this will make the formulas come out nicer and simpler. It is also OK to approximate and ignore small error terms if it simplifies the resulting formulas. Use the facts about birth/death chains learnt in class (see page 45 of scanned lecture notes).

3. Consider the Markov chain on state space  $\{1, 2, 3, 4, 5\}$  with transition matrix

$$\underline{\underline{P}} = \begin{pmatrix} 0 & 1/3 & 2/3 & 0 & 0 \\ 0 & 0 & 0 & 1/4 & 3/4 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

- (a) Show that the Markov chain is irreducible by drawing its accessibility graph.
- (b) What is the period of this Markov chain?
- (c) Find the stationary distribution  $(\pi_1, \dots, \pi_5)$  of the Markov chain.

*Hint:* First make a guess about the value of  $\pi_1$  using the law of large numbers for Markov chains.

- (d) Find  $\underline{\underline{P}}^{2018}$ .