PROBABILITY 1 | MIDTERM 1 RE-TAKE | 2025. 11. 05

1 a) $\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{4}$

b) $1 - \mathbb{P}($AT MOST ONE KNIGHT CROSSES SAFELY$) =$

$$= 1 - \left(\frac{3}{4}\right)^5 - \binom{5}{1} \cdot \left(\frac{3}{4}\right)^4 \cdot \left(\frac{1}{4}\right) = 0.367$$

c) $\mathbb{P}($SURVIVED THE FIRST Q $|$ DIED$) =$

$$= \frac{\mathbb{P}(\text{SURV. 1ST Q AND DIED})}{\mathbb{P}(\text{DIED})} = \frac{\frac{3}{4} \cdot \left(1 - \frac{2}{3} \cdot \frac{1}{2}\right)}{1 - \frac{1}{4}} = \frac{2}{3}$$

BONUS: $\cos\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{\pi}{16}\right) \cdot \ldots = ?$

$$1 = \sin\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{4}\right) \cdot \sin\left(\frac{\pi}{4}\right) \cdot 2 =$$

$$= \cos\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{8}\right) \cdot \sin\left(\frac{\pi}{8}\right) \cdot 2^2 =$$

$$= \cos\left(\frac{\pi}{4}\right) \cdot \cos\left(\frac{\pi}{8}\right) \cdot \cos\left(\frac{\pi}{16}\right) \cdot \sin\left(\frac{\pi}{16}\right) \cdot 2^3 = \ldots$$

$$= ? \cdot \lim_{n \to \infty} \sin\left(\frac{\pi}{2^n}\right) \cdot \frac{2^n}{2} \stackrel{\text{L'HOSPITAL}}{=}$$

$$= ? \cdot \frac{\pi}{2} \quad \text{THUS} \quad \boxed{? = \frac{2}{\pi}}$$

(2)

a) $\left(1-\frac{1}{7}\right)^{12}$

b) $A_i :=$ { DWARF i GETS AT LEAST ONE }

$$\mathbb{P}\left(\bigcap_{i=1}^{7} A_i\right) = 1 - \mathbb{P}\left(\bigcup_{i=1}^{7} A_i^c\right) = \bigstar$$

$$\mathbb{P}\left(\bigcup_{i=1}^{7} A_i^c\right) \overset{\text{INCL. EXCL.}}{=\!=} \sum_{\substack{I \subseteq [7] \\ I \neq \emptyset}} (-1)^{|I|+1} \cdot \mathbb{P}\left(\bigcap_{i\in I} A_i^c\right) = \ddot{\smile}$$

$$\mathbb{P}\left(\bigcup_{i\in I} A_i^c\right) = \left(1-\frac{|I|}{7}\right)^{12}$$

$$\ddot{\smile} = \sum_{r=1}^{7} (-1)^{r+1} \cdot \binom{7}{r} \cdot \left(1-\frac{r}{7}\right)^{12}$$

$$\bigstar = 1 - \ddot{\smile} = \sum_{r=0}^{7} (-1)^{r} \cdot \binom{7}{r} \cdot \left(1-\frac{r}{7}\right)^{12}$$