

Name: ..... Neptun code: ..... Tutor: .....

**Probability Theory 1 retaken 1st midterm, 5th November 2025.**

**16:15–17:00**

*Working time: 45 minutes. Only simple scientific, non-programmable calculators are allowed.*

*Maximum score (with the bonus exercise): 24 points, but we consider 20 points already as 100%.*

1. King Arthur and the knights of the round table (Sir Bedivere, Sir Galahad, Sir Lancelot, Sir Bors and Sir Robin) arrived to the Bridge of Death. The keeper of the bridge asks ~~five~~ three questions from each knight. Those who can answer the ~~five~~ three questions right may cross in safety, but who answers to a question wrong are cast into the Gorge of Eternal Peril. The questions are getting harder and harder, namely, each knight can answer the first question right with probability  $3/4$ , but the second question only with probability  $2/3$ , and the third only with probability  $1/2$ . The questionnaires of each knight are independent (they cannot help each other).

- (a) (2 points) What is the probability that a knight may cross in safety?
- (b) (3 points) What is the probability that at least two knights may cross in safety (King Arthur is not counted)?
- (c) (5 points) Sir Robin was cast into the Gorge of Eternal Peril. What is the probability that he was cast after the second question?

**Bonus:** (4 points) The keeper makes an exception with King Arthur, they play the following: They take turns asking each other and who gives the first wrong answer will be cast into the Gorge of Eternal Peril. First, the keeper asks then Arthur asks (if he gave the correct answer for the first question) then the keeper again etc. The probability that the  $n$ th question is answered correctly is  $\cos(\frac{\pi}{2^{n+1}})$ . What is the probability that no one will be cast into the Gorge of Eternal Peril and they keep asking each other forever? (Explicit number is needed.) (*Hint:  $\sin(2x) = 2 \sin(x) \cos(x)$ .*)

2. Snow White decides to surprise the seven dwarfs, and she puts 12 little apples below their pillows. But she distributes the apples randomly, so she puts each apple independently under the pillow of a uniformly randomly chosen dwarf.
- (a) (3 points) What is the probability that Dopey does not get any apple under his pillow?
  - (b) (7 points) What is the probability that every dwarf gets at least one apple?