VSZ1 | ZH 2 | 2025.11.24 | A

① $B \sim N(7.00, 12^2)$ $C \sim N(7.05, 5^2)$

CLYDE BOSSZÚS, HA $B > C + 7$

BONNIE SÉRTŐDÖTT, HA $C > B + 4$

$P(\text{RANDEVÚ OK}) = P(B \leq C+7, C \leq B+4) =$

$= P(-7 \leq C - B \leq 4) =$ ☺

$D := C - B$ $D \sim N(5, \underbrace{12^2 + 5^2}_{13^2})$

☺ $= P(-7 \leq D \leq 4) = P\left(\frac{-7-5}{13} \leq \frac{D-5}{13} \leq \frac{4-5}{13}\right) =$

$= \Phi\left(\frac{-1}{13}\right) - \Phi\left(\frac{-12}{13}\right) = \left(1 - \Phi\left(\frac{1}{13}\right)\right) - \left(1 - \Phi\left(\frac{12}{13}\right)\right)$

$= \Phi\left(\frac{12}{13}\right) - \Phi\left(\frac{1}{13}\right) = 0.8212 - 0.5319 = 0.2893$

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② $G(x) = P(X \leq x) = \begin{cases} 0 & \text{HA } x < 0 \\ ★ & \text{HA } 0 < x < 1 \\ 1 & \text{HA } x > 1 \end{cases}$

★ $= P(e^{-\xi} \leq x) = P(-\xi \leq \ln(x)) =$

$P(\xi \geq -\ln(x)) =$ ☹

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(ii) $= \mathbb{P}\left(\xi \geq -\ln(x)\right) = e^{-2\cdot(-\ln(x))} = x^2$

$$g(x) = G'(x) = \begin{cases} 0 & \text{HA } x<0 \\ 2x & \text{HA } 0\leq x\leq 1 \\ 0 & \text{HA } x>1 \end{cases}$$

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b) $\mathbb{P}\left(X<\frac{1}{4} \mid X<\frac{1}{2}\right) = \frac{\mathbb{P}\left(X<\frac{1}{4}\right)}{\mathbb{P}\left(X<\frac{1}{2}\right)} = \frac{(1/4)^2}{(1/2)^2} =$

$= \frac{1/16}{1/4} = \frac{1}{4}$ (BIZTOS KIZÖTT VOLNA AZ EXPONENCIÁLIS EO. ÖRÖKIFJÚSÁGÁBÓL IS)

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(3) $\eta_i := \mathbb{I}[A_i]$, $A_i = \{ i \text{ KIUGRÓ} \}$

$\eta = \sum_{i=2}^{99} \eta_i$ $\quad \mathbb{E}(\eta) = 98 \cdot \mathbb{P}(A_2) = 98 \cdot \frac{1}{4} = 24{,}5$

HISZ $\mathbb{P}(A_2) = \mathbb{P}(FIF) + \mathbb{P}(IFI) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$\mathrm{Var}(\eta) = \sum_{i,j=2}^{99} \underbrace{\mathrm{Cov}(\eta_i, \eta_j)}_{C_{ij}} = \boxed{\ddot{\smile}}$

$\boxed{i=j}$: $C_{ii} = \frac{1}{4}\cdot\left(1-\frac{1}{4}\right) = 3/16$

$\boxed{|i-j|\geq 2}$: $C_{ij} = 0$ (FÜGGETLENSÉG MIATT)

$\boxed{|i-j|=1}$: $C_{i,i+1} = C_{i,i-1} = \left(\mathbb{P}(FIFI) + \mathbb{P}(IFIF)\right) - \left(\frac{1}{4}\right)^2 =$

$= \frac{1}{16}$, THUS $\boxed{\ddot{\smile}} = 98\cdot\frac{3}{16} + 2\cdot 97\cdot\frac{1}{16} = 30{,}5$

$\sqrt{\mathrm{Var}(\eta)} = 5{,}52$

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