VSZ1 VIZSGA 2024.12.10.

$\boxed{\text{GYAK 1}}$

a) $A_i := \{\text{TALÁLOK } i\text{-EDIK FAJTA CSOKIT}\}$ $i=1,2,3,4$

$$\mathbb{P}\left(\bigcap_{i=1}^{4} A_i\right) = 1 - \mathbb{P}\left(\bigcup_{i=1}^{4} A_i^c\right) \overset{\text{SZITA}}{=\!=\!=}$$

$\boxed{\text{HISZ VALAMILYEN CSOKIT TALÁLOK}}$

$$1 - \sum_{i=1}^{4} \mathbb{P}(A_i^c) + \sum_{i<j} \mathbb{P}(A_i^c \cap A_j^c) - \sum_{i<j<k} \mathbb{P}(A_i^c \cap A_j^c \cap A_k^c) + 0$$

$$= 1 - 4 \cdot \frac{\binom{14}{6}}{\binom{24}{6}} + 6 \cdot \frac{\binom{14}{12}}{\binom{24}{12}} - 4 \cdot 0 \approx 0.91$$

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b) $X_i := \mathbb{1}[B_i]$ $B_i = \{i\text{-EDIK NAP ÉTCSOKI}\}$

$Y_i := \mathbb{1}[C_i]$ $C_i = \{\text{-"- NUGÁT}\}$

$X = \sum_{i=1}^{10} X_i$ = DECEMBER 10-IG ÉTCSOKI SZÁM

$Y = \sum_{i=1}^{10} Y_i$ = -"- -"- NUGÁT SZÁM

$$\mathrm{Cov}(X,Y) = \sum_{i,j=1}^{10} \mathrm{Cov}(X_i, Y_j) = \sum_{i,j=1}^{10} \mathbb{P}(B_i \cap C_j) - \left(\frac{1}{4}\right)^2 = ⊛$$

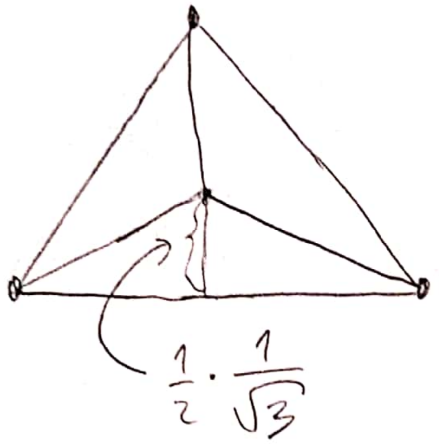
$\boxed{i=j}$ $\mathbb{P}(B_i \cap C_j) = \mathbb{P}(B_i \cap C_i) = 0$

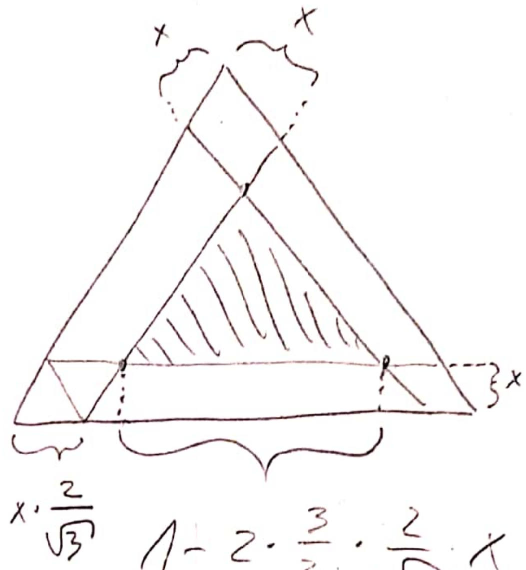
$\boxed{i \neq j}$ $\mathbb{P}(B_i \cap C_j) = \mathbb{P}(B_i) \cdot \mathbb{P}(C_j | B_i) = \frac{1}{4} \cdot \frac{6}{23}$

$$⊛ = 10 \cdot \left(-\left(\frac{1}{4}\right)^2\right) + 90 \cdot \left(\frac{1}{4} \cdot \left(\frac{6}{23} - \frac{1}{4}\right)\right) = -\frac{35}{92} = -0.38\ldots$$

$\boxed{\text{GYAK 2}}$

$$T = \frac{\sqrt{3}}{4}$$





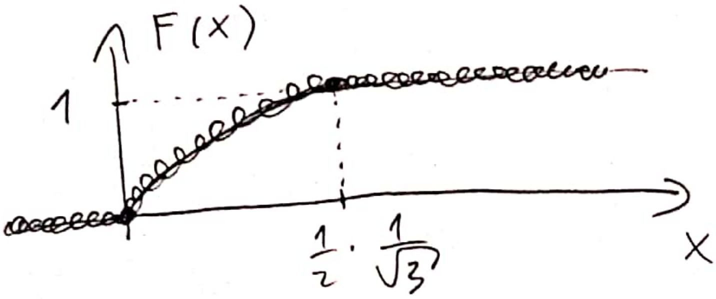
$$1 - 2\cdot\frac{3}{2}\cdot\frac{2}{\sqrt{3}}\, x =$$

$$= 1 - 2\cdot\sqrt{3}\cdot x$$

$$F(x) = \begin{cases} 0, & \text{HA } x \le 0 \\ \star, & \text{HA } x \in \left(0, \frac{1}{2}\cdot\frac{1}{\sqrt{3}}\right) \\ 1, & \text{HA } x \ge \frac{1}{2}\cdot\frac{1}{\sqrt{3}} \end{cases}$$

$\star = 1 - \mathbb{P}(\text{SATÍROZOTT } \Delta\text{-BE ESIK A PONT}) =$

$$= 1 - \frac{(1-2\cdot\sqrt{3}\cdot x)^2\cdot\frac{\sqrt{3}}{4}}{\sqrt{3}/4} = 1 - (1 - 2\cdot\sqrt{3}\cdot x)^2$$



$\sqrt{3}/6$

$$f(x) = F'(x) = 2\cdot(1 - 2\cdot\sqrt{3}\cdot x)\cdot 2\cdot\sqrt{3}\cdot \mathbb{I}\left[0 < x < \frac{1}{2\cdot\sqrt{3}}\right]$$

$$= (4\cdot\sqrt{3} - 24\cdot x)\cdot\mathbb{I}\left[0 < x < \frac{1}{2\cdot\sqrt{3}}\right]$$

GYAK 3

$$S_{36} = X_1 + \dots + X_{64} = \text{ELSŐ VIZSGA ÖSSZ-PONTSZÁMA}$$

$$S^*_{49} = X^*_1 + \dots + X^*_{49} = \text{MÁSODIK} \;-''-\;-''-\;-''-$$

$$Y := \frac{S_{36} - 36\cdot 74}{\sqrt{36}\cdot 14} \overset{\text{C.H.T.}}{\approx} N(0,1) \overset{\text{C.H.T.}}{\approx} Y^* = \frac{S^*_{49} - 49\cdot 74}{\sqrt{49}\cdot 14}$$

a) $$\mathbb{P}\left(\left|\frac{S_{36}}{36} - \frac{S^*_{49}}{49}\right| \geq 2\right) =$$

$$\mathbb{P}\left(\left|\frac{6\cdot 14\cdot Y + 36\cdot 74}{36} - \frac{7\cdot 14\cdot Y^* + 49\cdot 74}{49}\right| \geq 2\right) =$$

$$= \mathbb{P}\left(\left|\frac{14}{6}\cdot Y - \frac{14}{7}\cdot Y^*\right| \geq 2\right) = (\star)$$

$$\boxed{Z_1 := \frac{14}{6}\cdot Y - \frac{14}{7}\cdot Y^*} \qquad Z_1 \sim N\Big(0, \underbrace{\left(\tfrac{14}{6}\right)^2 + \left(\tfrac{14}{7}\right)^2}_{=\sigma^2}\Big)$$

AHOL $\boxed{\sigma = 3.073}$

$$(\star) = \mathbb{P}(|Z| \geq 2) = 2\cdot\left(1 - \Phi\left(\frac{2}{\sigma}\right)\right)$$

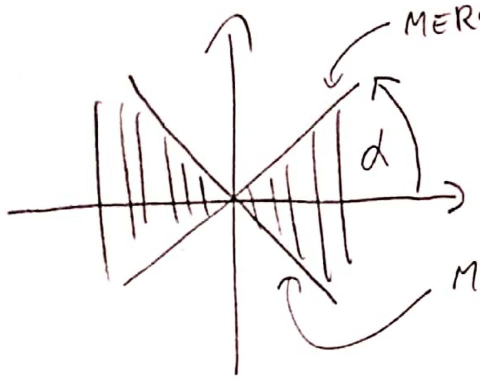
$$= 2\cdot(1 - \Phi(0.65)) = 2\cdot(1 - 0.7422)$$

$$= 0.5156$$

$\boxed{\text{GYAK 3}}$ b) $\mathbb{P}\left(\left|\frac{S_{49}^*}{49}-74\right| < \left|\frac{S_{36}}{36}-74\right|\right) =$

$= \mathbb{P}\left(\left|\frac{7\cdot 14\cdot Y^*}{49}\right| < \left|\frac{6\cdot 14\cdot Y}{36}\right|\right) = \mathbb{P}\left(\left|\frac{Y^*}{7}\right| < \left|\frac{Y}{6}\right|\right) =$

$= \mathbb{P}\left(|Y^*| < \frac{7}{6}\cdot|Y|\right) = \mathbb{P}\left((Y, Y^*) \in \text{SATÍR}\right) =$



$= \frac{4\alpha}{2\pi} = \frac{2\alpha}{\pi} =$

$\frac{2}{\pi}\cdot \arctan\left(\frac{7}{6}\right) =$

0.54887

$\boxed{\text{ELM 1}}$ a) $\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A\cap B)}{\mathbb{P}(B)}$, HA $\mathbb{P}(B) \neq 0$

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b) $\mathbb{P}(E_1\cap\ldots\cap E_n) = \mathbb{P}(E_1)\cdot\mathbb{P}(E_2\mid E_1)\cdot\ldots\cdot\mathbb{P}(E_n\mid E_1\cap\ldots\cap E_{n-1})$

<u>BIZ</u>: $\mathbb{P}(E_1)\cdot\frac{\mathbb{P}(E_2\cap E_1)}{\mathbb{P}(E_1)}\cdot\ldots\cdot\frac{\mathbb{P}(E_n\cap E_{n-1}\cap\ldots\cap E_1)}{\mathbb{P}(E_1\cap\ldots\cap E_{n-1})} =$

$\underset{\uparrow}{=} \mathbb{P}(E_1\cap\ldots\cap E_n)$

TELESZKOPIKUS

$\boxed{\text{ELM 1}}$ c)

$E_i := \{$ AZ ELSŐ i ÁSZ MIND KÜLÖN JÁTÉKOSHOZ JUT $\}$

$E_4 \subseteq E_3 \subseteq E_2 \subseteq E_1 = \Omega$

$\mathbb{P}($MIND A 4 JÁTÉKOS 1-1 ÁSZT KAP$) = \mathbb{P}(E_4) =$

$$= \underbrace{\mathbb{P}(E_1)}_{1} \cdot \underbrace{\mathbb{P}(E_2 | E_1)}_{\frac{3 \cdot 13}{51}} \cdot \underbrace{\mathbb{P}(E_3 | E_2)}_{\frac{2 \cdot 13}{50}} \cdot \underbrace{\mathbb{P}(E_4 | E_3)}_{\frac{13}{49}} \approx 0.1$$

$\boxed{\text{ELM 2}}$ a) HA $X \geq 0$, AKKOR $\mathbb{P}(X \geq k) \leq \frac{\mathbb{E}(X)}{k}$
ÉS $k \in \mathbb{R}_+$

BIZ: $\mathbb{E}(X) \geq \mathbb{E}(X \cdot \mathbb{1}[X \geq k]) \geq \mathbb{E}(k \cdot \mathbb{1}[X \geq k])$
$= k \cdot \mathbb{P}(X \geq k)$

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b) HA $\mathbb{E}(X) = m$, AKKOR $\mathbb{P}(|X - m| \geq k) \leq \frac{Var(X)}{k^2}$

BIZ: $\boxed{Y := X - m}$ $\boxed{\mathbb{E}(Y^2) = Var(X)}$

$$\mathbb{P}(|X - m| \geq k) = \mathbb{P}(Y^2 \geq k^2) \overset{\text{MARKOV}}{\leq} \frac{\mathbb{E}(Y^2)}{k^2}$$

5. OLDAL

$\boxed{\text{ELM 2}}$ c) $Y = U_1 + \ldots + U_6$

$$\mathbb{E}(Y) = 6 \cdot \mathbb{E}(U_1) = 6 \cdot \frac{1}{2} = 3$$

$$\mathrm{Var}(Y) = 6 \cdot \mathrm{Var}(U_1) = 6 \cdot \frac{1}{12} = \frac{1}{2}$$

$$\mathbb{P}(|Y-3| \geq 2) \leq \frac{\mathrm{Var}(Y)}{2^2} = \frac{1}{8}$$

$\boxed{\text{ELM 3}}$ a) $\mathbb{P}(X = k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$, $k = 0, 1, 2, \ldots$

b) $$M(t) = \sum_{k=0}^{\infty} e^{t \cdot k} \cdot e^{-\lambda} \cdot \frac{\lambda^k}{k!} = e^{-\lambda} \cdot \sum_{k=0}^{\infty} \frac{(e^t \cdot \lambda)^k}{k!} =$$

$$= e^{-\lambda} \cdot \exp(e^t \cdot \lambda) = \exp((e^t - 1) \cdot \lambda)$$

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c) $$\sum_{k=0}^{\infty} e^{-\lambda} \cdot \frac{\lambda^k}{k!} = M(0) = \exp((e^0 - 1) \cdot \lambda) = \exp(0) = 1$$

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d) $\mathbb{E}(X) = M'(0) = \ldots = \lambda$

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e) $\mathrm{Var}(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = M''(0) - \lambda^2 = \ldots = \lambda$

<u>BÓNUSZ</u>: LÁSD GYAK 5. FELADATSOR „POISSON HANDOUT"