STATISTICAL MECHANICS AND BILLIARDS. GRADUATE COURSE.

DOMOKOS SZÁSZ

BUDAPEST UNIVERSITY OF TECHNOLOGY

The basic philosophy of statistical physics is that its macroscopic laws arise from microscopic principles of newtonian mechanics. For their derivation billiard models have become most successful. These billiards are hyperbolic dynamical systems with singularities whose mathematical theory, however, is quite involved. Therefore - rather than treating hyperbolic billiards in general - my goal in this course is twofold: on the one hand, I explain parts of their 'dynamical systems' theory on simple paradigm models and, on the other hand, I will deal with some probabilistic methods which can be applied or are mimicked when deriving the macroscopic laws for billiard type models. At each topic, I will also formulate recent results and open problems.

Topics to be treated will be selected from the following ones:

- Proving ergodicity for hyperbolic maps: the Hopf method for the algebraic automorphism of the torus (Arnold's cat map)
- Proving ergodicity for hyperbolic maps with singularities: The fundamental theorem for hyperbolic billiards for the singular version of Arnold's cat map.
- A ball-avoiding theorem and ergodicity for a semi-dispersing billiard; a sketch
- Controlled mixing and CHT for hyperbolic maps with singularities: Young's towers and Perron-Frobenius operator technique for the singular version of Arnold's cat map
- Local CLT for the singular version of Arnold's cat map
- Markov maps of the interval: existence of absolutely continuous invariant measure
- CLT for the locally perturbed simple symmetric random walk in dimension 2: bounded range (coupling method)
- CLT for the locally perturbed simple symmetric random walk in dimension 2: unbounded range
- Probability theory of random walks with internal states
- A mechanical model of Brownian motion: the Rayleigh gas