STATISTICAL MECHANICS AND BILLIARDS.
GRADUATE COURSE. SYNOPSIS.

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Reminder (Notes in Hungarian). [Sz2]

- Endomorphism, automorphism w/wout invariant measure. Flows.
- Poincaré recurrence thm.
- Examples.
- Ergodic thm's: Neumann, Birkhoff-Khintchin.
- Ergodicity. R_α is ergodic if $\alpha \notin \mathbb{Q}$.
- Mixing.
- Symbolic dynamics. Bernoulli endo's/auto's are mixing. Isomorphism.

Topics.

(1) Algebraic auto of torus. NSC for their being mixing (via characters). Hyperbolicity of algebraic auto of torus. Hopf method. (cf. [Sz2])
(2) Suspension and cross-section (cf. [BS]).
(3) Billiards. Billiard ball map, flow.
(4) Examples of billiards.
(5) Hopf method for the singular version of algebraic auto of torus. I. Existence of smooth invariant manifolds (**Notes**).
(6) (without proofs) Anosov maps, hyperbolic sets of diffeos. (Hadamard-Perron theorem. Local product structure. Markov-partition.)
(7) Subshifts, subshifts of finite type. Semi-conjugacy, isomorphy. (cf. [BS])
(8) Markov partition for Arnold's cat. Sinai's (1968) iterative construction. A simple construction à la Adler-Weiss (cf. [BS]).
(9) Markov property of the symbolic dynamics. A CLT for simple functions by using martingale CLT (and CLT for finite Markov chains; cf. Sethuraman: A martingale CLT, http://orion.math.iastate.edu/sethuram/papers/wi_mart.pdf).
(10) Billiards. Collision equations for semi-dispersing billiards (**Notes**). Differential of the collision map. Invariant measure. Evolution of wavefronts. (all for $d = 2$) (cf. [ChM]).

(11) Hyperbolicity of dispersing billiards. Non-smoothness of the dynamics. Definition of local invariant manifolds. Continued fraction form of their curvature (cf. [ChM] and [KSSz2]).
(12) Hopf method for the singular version of algebraic auto of torus. II. Fundamental thm, zigzag thm and local ergodicity thm (**Notes**).
(13) Multidimensional semi-dispersing billiards (cf. [KSSz2]).
(14) Local ergodicity theorem for semi-dispersing billiards (without proof). Ergodicity of dispersing billiards. (cf. [KSSz2, Sz1])
(15) Ergodicity of a semi-dispersing billiard. A ball-avoiding theorem. (cf. [KSSz2, Sz1])
(16) Markov maps of the interval. Perron-Frobenius operator, existence of acim. (cf. [CE]).
(17) Controlled mixing and CHT for hyperbolic maps with singularities: Young's towers and Perron-Frobenius operator technique for the singular version of Arnold's cat map (cf. [Y],).
(18) Local CLT for random walks with internal states (cf. [KSz]).

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