Differential Equations 1. homework

Deadline: October 04, 2018. On the practical lecture in paper format or send your solution to szokemarton3@gmail.com.

1. exercise Find the general solution of the following differential equation, where $x > 0$, then solve the initial value problem, where $y(\pi) = 0$.

$$xy' = \cos x - 2y$$

2. exercise Find the orthogonal trajectories of the curves $y - 1 = cx^3$ (c is an arbitrary constant). Sketch both families of curves.

3. exercise Solve the $y' = \frac{2x+y}{y-x}$ differential equation, where $x > 0$.

4. exercise Solve the following differential equation.

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{y^3}{x^4} + y}{\frac{y^2}{x^3} - x}$$

5. exercise Use the Euler method to find the approximation of the solution of the following initial value problem. Let the step-size $dx = 0.5$. Take at least 5 steps. To check, solve also the initial value problem.

$$y' = (y^2 + 1)x$$

$$y(-1) = 1$$

6. exercise Give the first 4 approximation of Picard's successive method for the initial value problem. Solve also the problem.

$$y' = y(2 + x^2)$$

$$y(0) = 1$$

7. exercise Solve this diffential equation, then check your solution by substituting it in the original equation.

$$5(1 + x^2)y' = 2xy + \frac{(1 + x^2)^2}{y^4}$$

8. exercise Take the $y' = y^2 - y - 6$ differential equation. Determine the fixed point, their type and the inflection points. Draw the phase line and some integral curves. Give also the analytic solution.

9. exercise Give the general solution of the following differential equation.

$$y'' - 4y' - 12y = 2e^{-2x} - 13e^{x}\cos x + 36x$$

10. exercise Find the solution of the following initial value problem with Newton's method.

$$y'' - y + 2e^{-x} = 0$$
$$y(0) = 0$$
$$y'(0) = 1$$