Differential Equations 2. homework

Deadline: November 15, 2018. On the practical lecture in paper format or send your solution to szokemarton3@gmail.com.

1. exercise Find the solution of the initial value problem with Laplace transformation.

$$y'' - y' - 12y = 2 + e^{2x}$$
$$y(0) = 1$$
$$y'(0) = 2$$

2. exercise Solve the following initial value problem.

$$y'' + 2x(y')^2 = 0$$
$$y(0) = 1$$
$$y'(0) = 4$$

3. exercise Solve the following initial value problem.

$$(1-x)y'' + xy' - y = 0$$
$$y(0) = 2$$
$$y'(0) = -1$$

Hint: Try to guess a solution of the equation.

4. exercise Find the general solution of the following differential equation.

$$y'' - \frac{y'}{x} - 3\frac{y}{x^2} = 1 + \frac{1}{x}$$

5. exercise Give the lowest order linear homogeneous differential equation with real constant coefficients, for which $y = 2\cosh x + 3xe^{-2x}\sin x$ is a solution.

6. exercise Find the general solution of the following system of differential equations by variation of parameters.

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$$

7. exercise Give the general solutions of the following system of differential equations in the cases a = -4; 0; +0.25 and +4, and draw the phase portraits in 3 different cases. Determine the type of the fixed points.

$$\dot{x} = -x + ay + 1$$
$$\dot{y} = x - y - 1$$

8. exercise Find the fixed points of the following system of differential equations, and sketch the phase portrait.

$$\dot{x} = 2x - xy$$
$$\dot{y} = x^2 - 2 - y$$

9. exercise Draw the phase portrait of the following system of differential equations.

$$\dot{x} = -y - xy^2 - x^3$$
$$\dot{y} = x - y^3 - x^2 y$$

Hint: Use polar coordinates, then multiply one of the equation by $\cos\varphi$ and the other one by $\sin\varphi$. After that sum the two new equations and subtract one of them from the other to get an equation for $\dot{r}$ and one for $\dot{\varphi}$.

10. exercise Write down and solve the variational system of the following initial value problem.

$$\dot{x} = e^y - 1$$
$$\dot{y} = 2x$$
$$x_0 = x(0) = 0$$
$$y_0 = y(0) = 0$$