Random Degree Constrained Process: Local Limit, Its Applications and Some Open Questions

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November 17, 2025

Kutszem,
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Joint work with Balázs Ráth and Lutz Warnke

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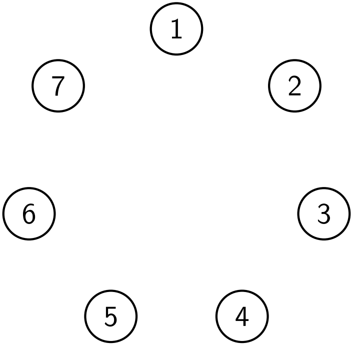
- Vertex v is **saturated** at time t if its degree in $G_{\underline{p}}^n(t)$ is equal to $d(v)$

An example for RDCP

RDCP on K_7 with degree constraint 3 for each vertex:



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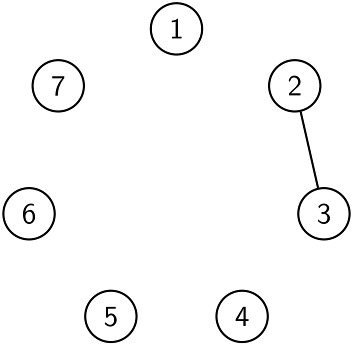

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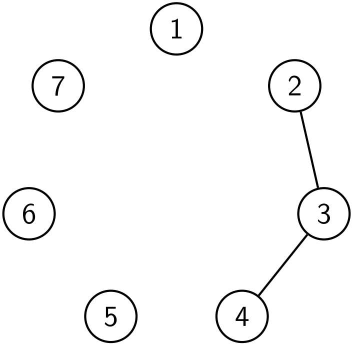

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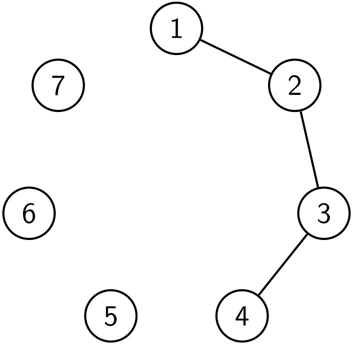

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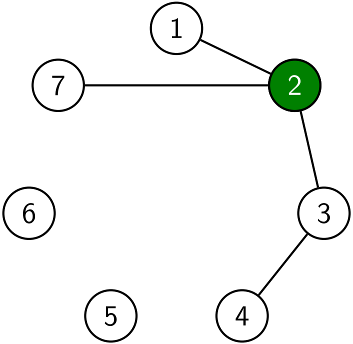

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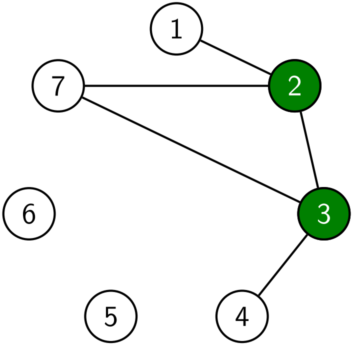

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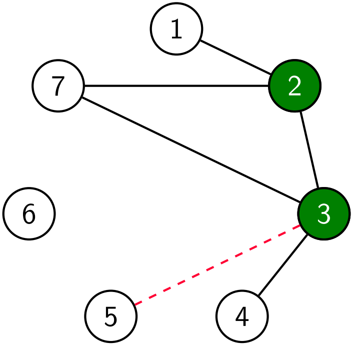

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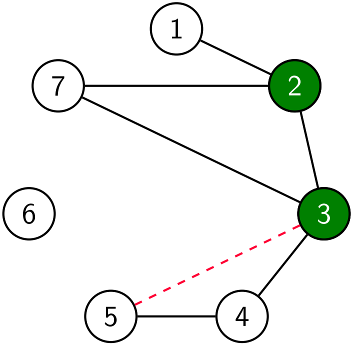

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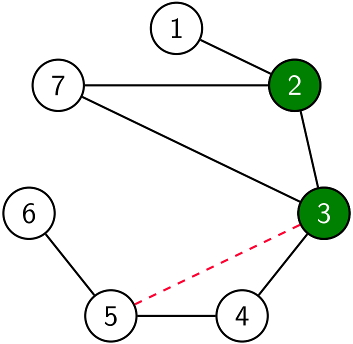

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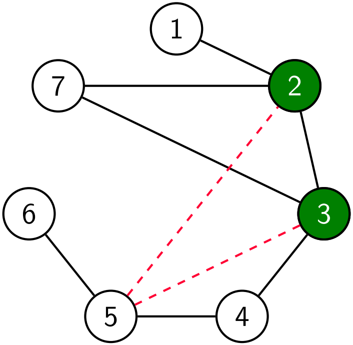

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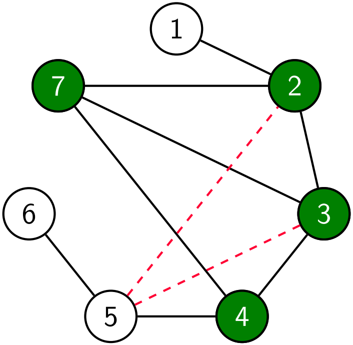

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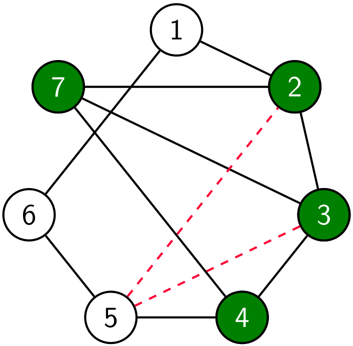

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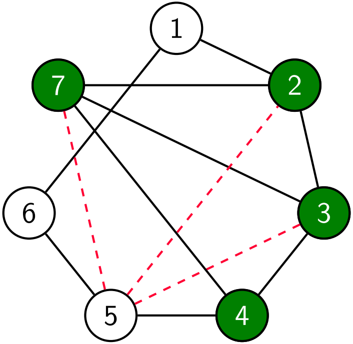

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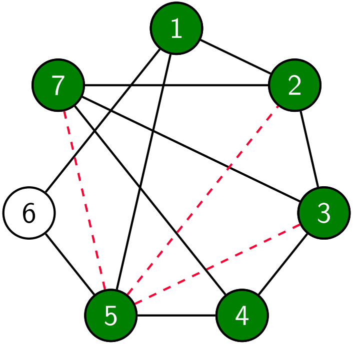

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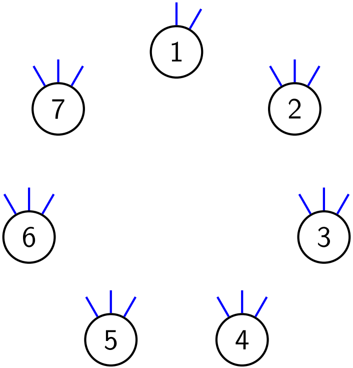

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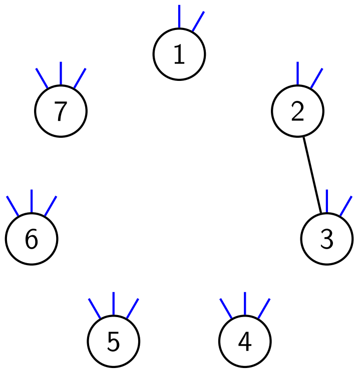

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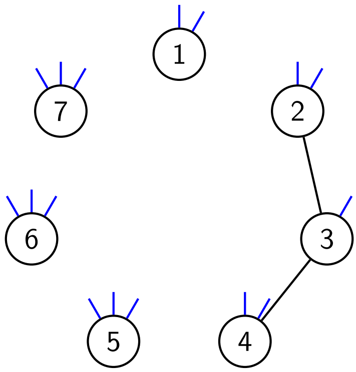

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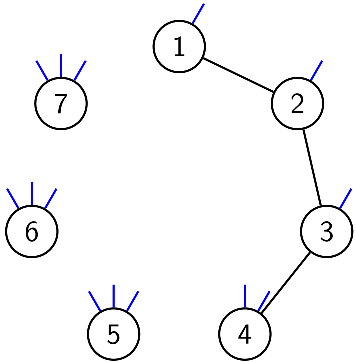

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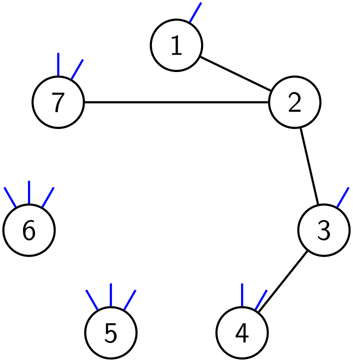

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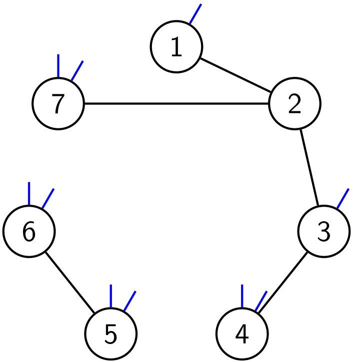

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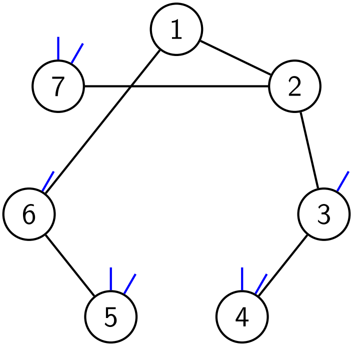

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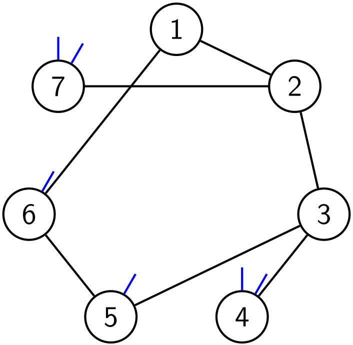

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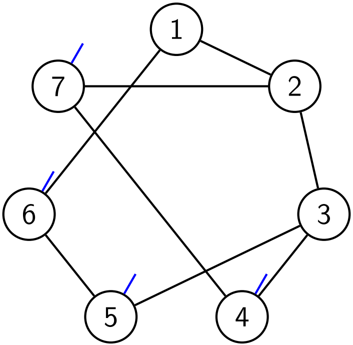

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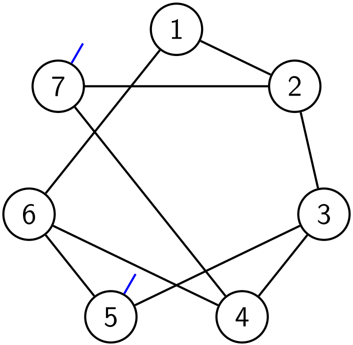

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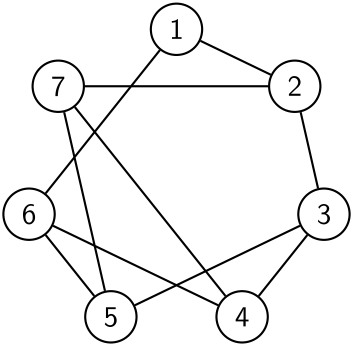

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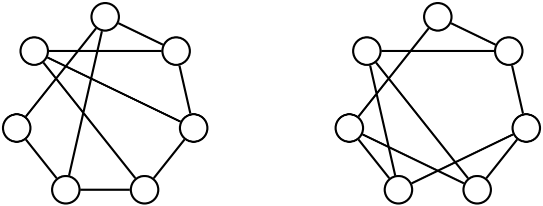

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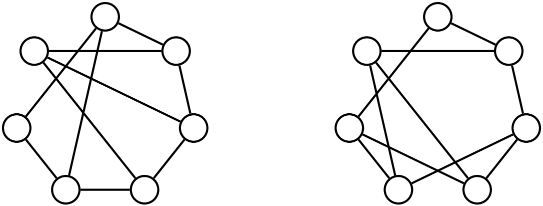

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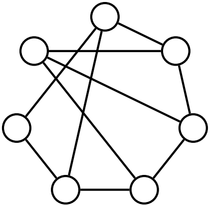

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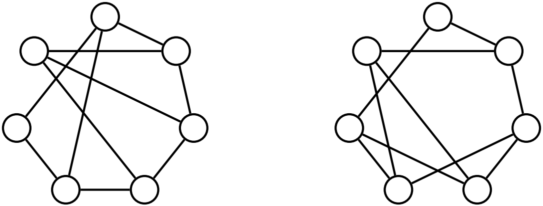

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- Open question (Wormald, 1999): Does this hold for the regular case?

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where $\mathcal{B}^R(u)$ is the R-neighbourhood of the root u in $\mathcal{G}$.

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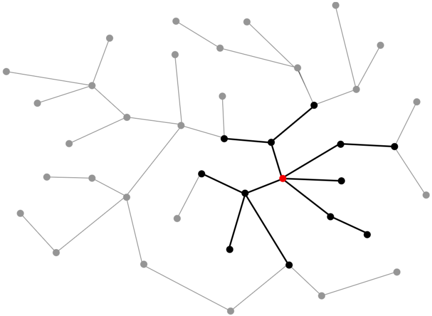

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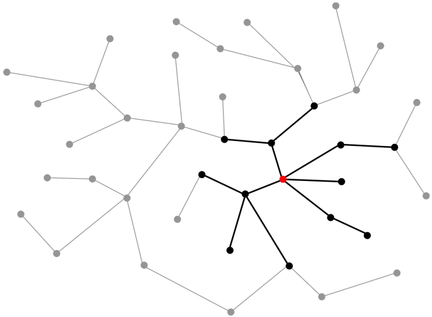

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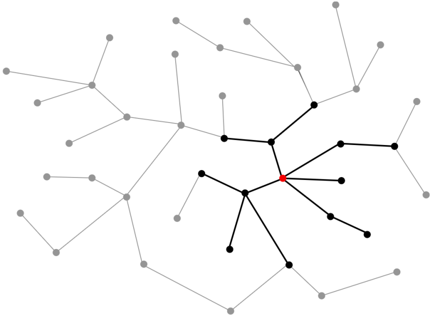

- It can be used to investigate numerous graph properties, parameters
- The limiting objects (often branching processes) are much simpler to study than the original model

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In contrast to the configuration model, the **local weak limit of the RDCP was unknown** before our work

Local weak limit of the RDCP

Theorem (Local weak convergence of RDCP (Ráth, Sz., Warnke, 2024+))

Fix $t \in \mathbb{R}_+ \cup \{\infty\}$. Then the random degree constrained process on K_n at time t converges locally to a random rooted tree $\left(G_{\underline{p}}^{\infty}(t), \varnothing\right)$, i.e.,

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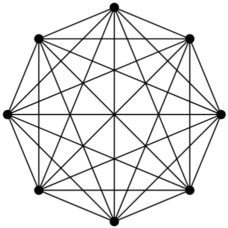

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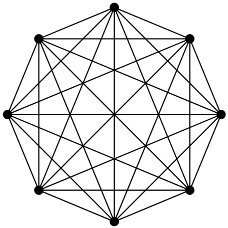

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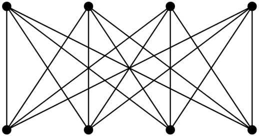

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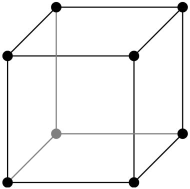

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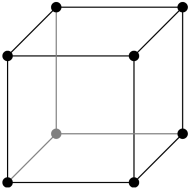

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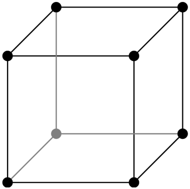

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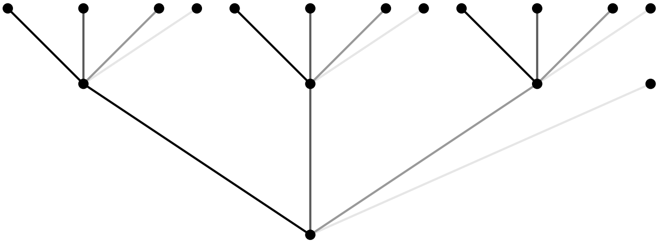

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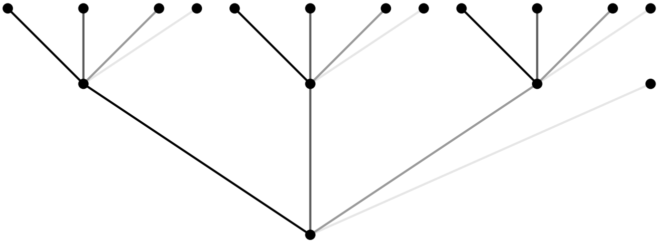

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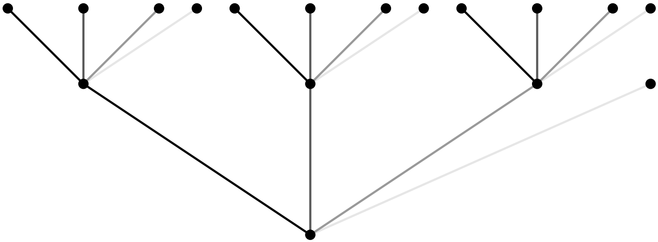

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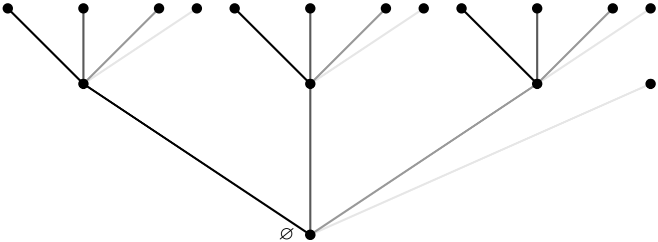

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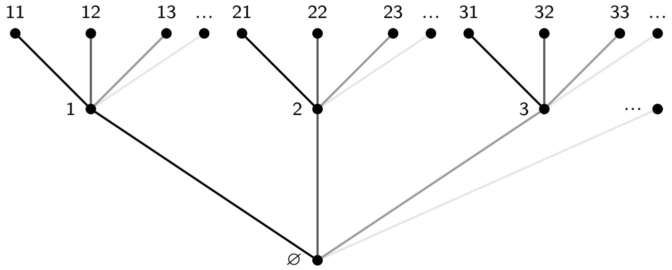

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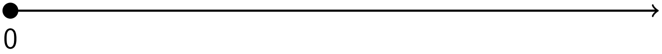

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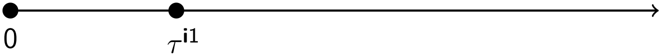

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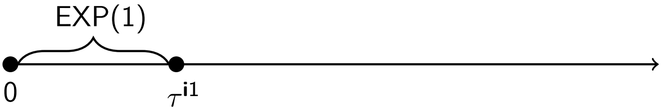

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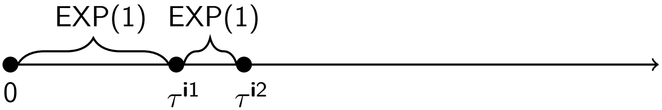

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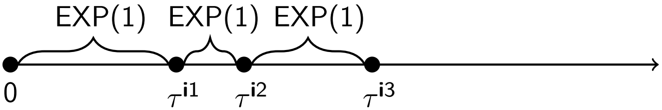

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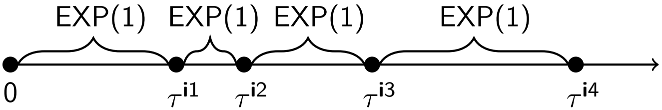

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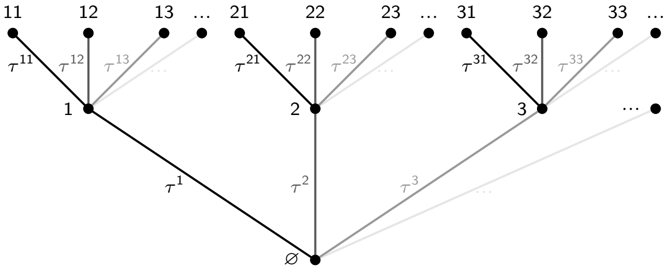

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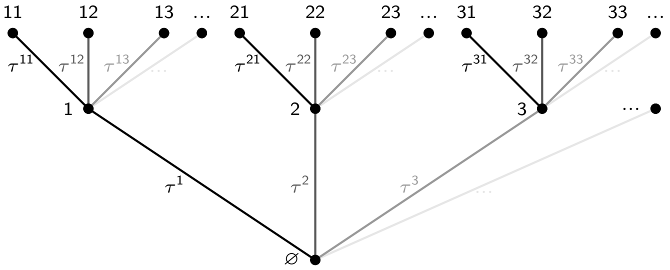

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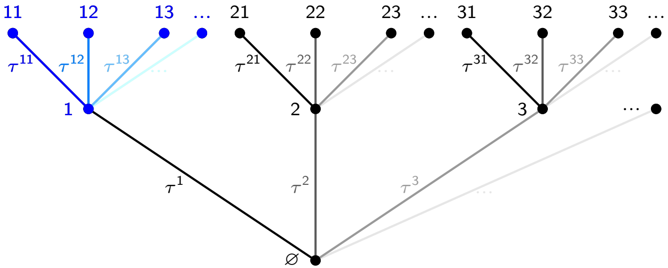

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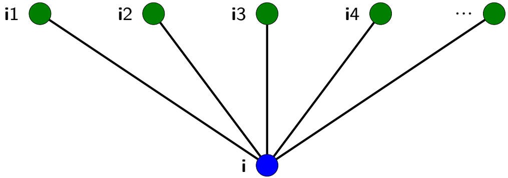

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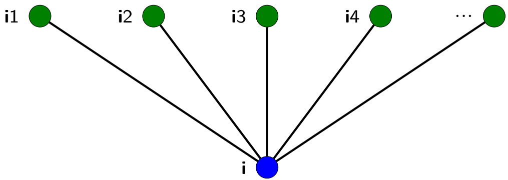

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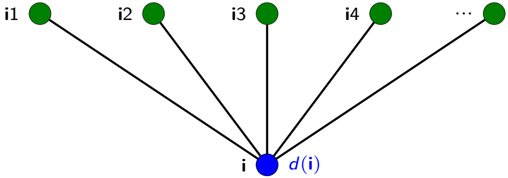

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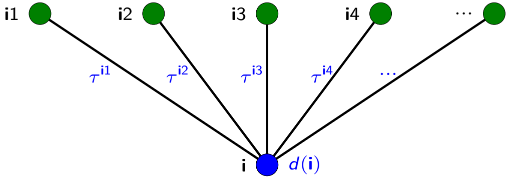

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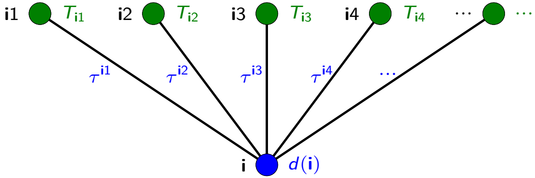

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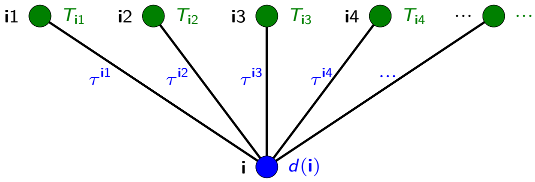

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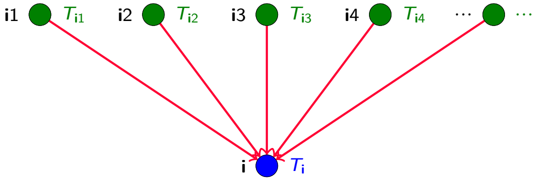

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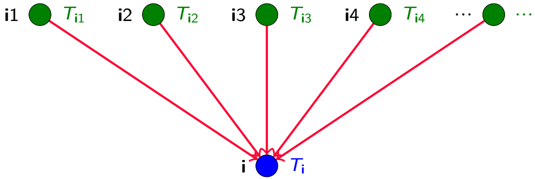

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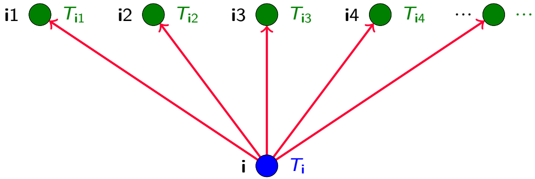

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- Other recursion for the distribution of phantom saturation times: generation r, $T_{\mathbf{i}}$ $\Rightarrow$ generation $(r+1)$, $(T_{\mathbf{i}j})_{j=1}^{\infty}$

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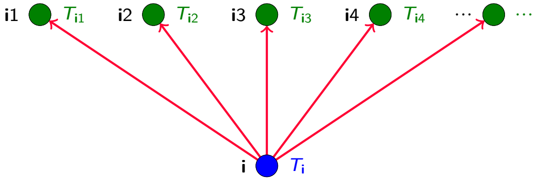

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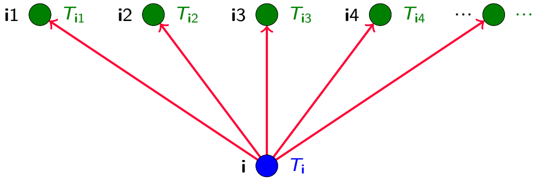

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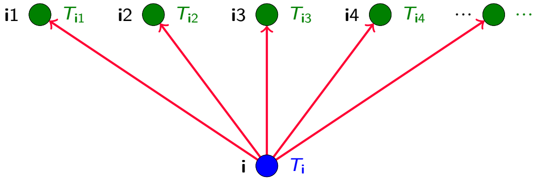

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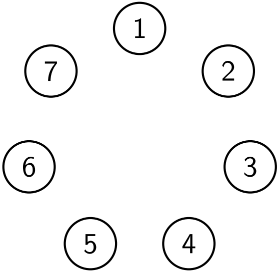

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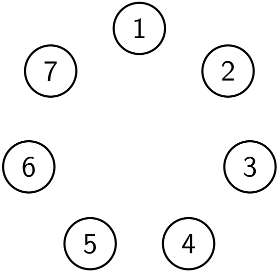

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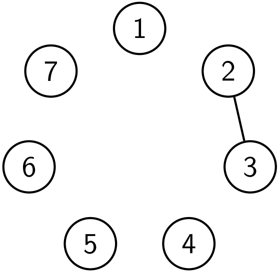

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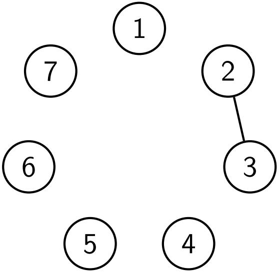

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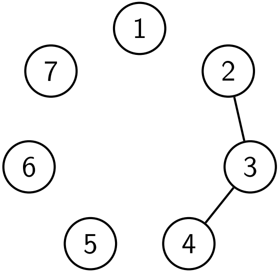

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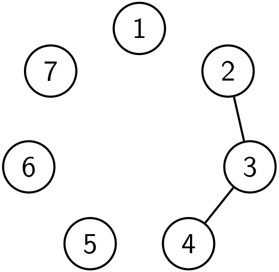

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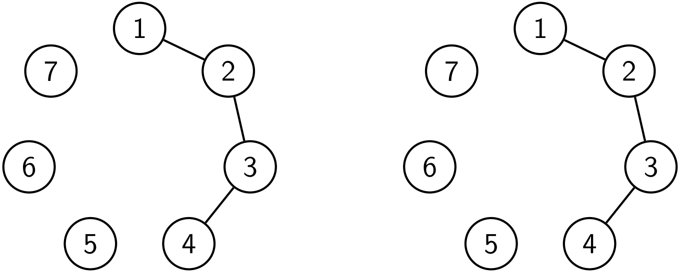

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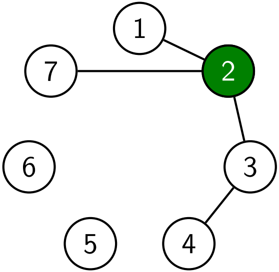

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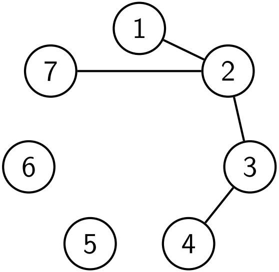

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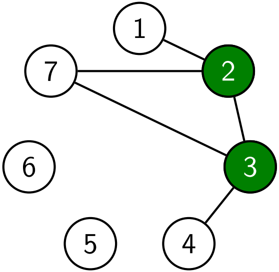

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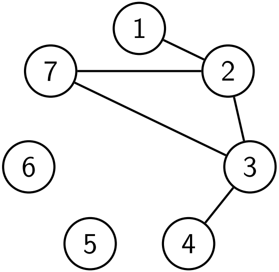

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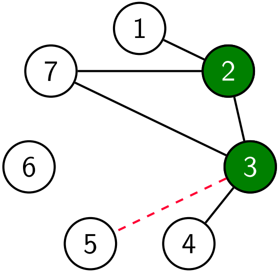

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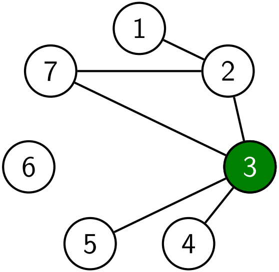

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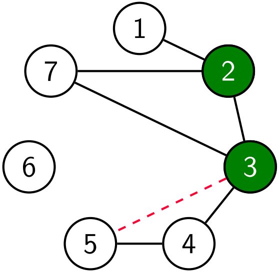

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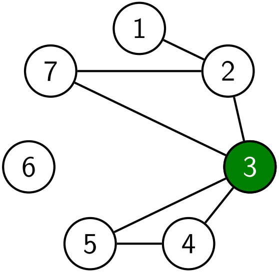

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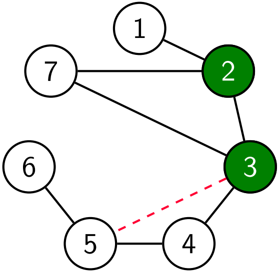

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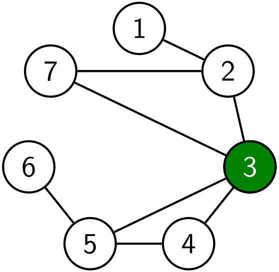

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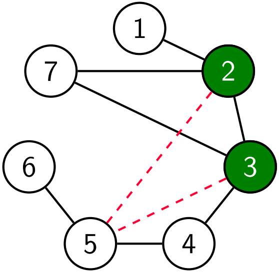

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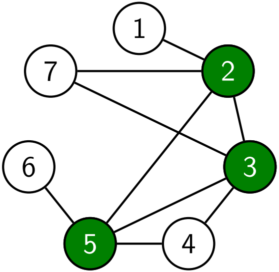

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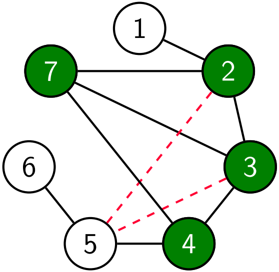

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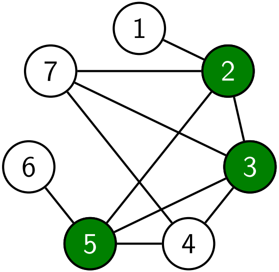

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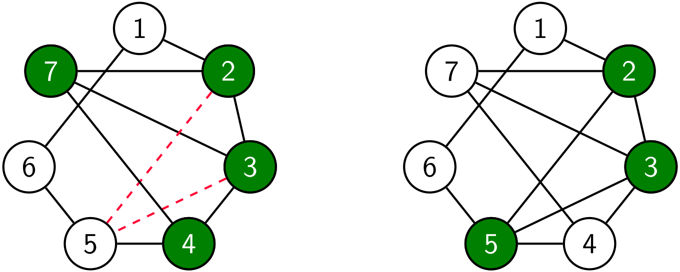

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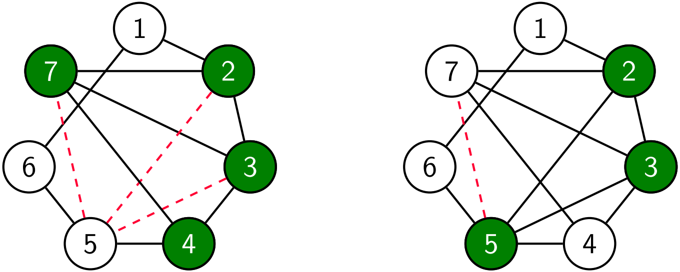

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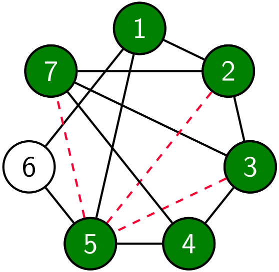

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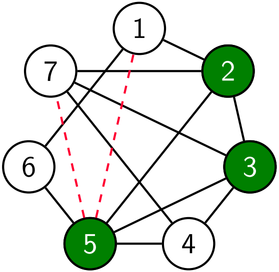

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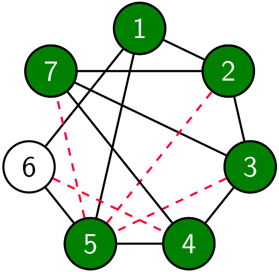

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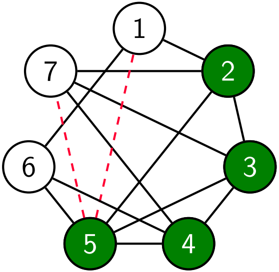

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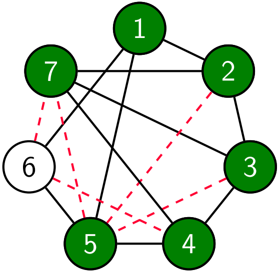

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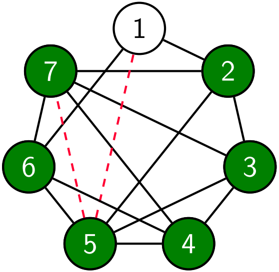

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  Namely: $\hat{t}_c$ can be defined as the first time when the local weak limit has infinitely many vertices with positive probability:

$$\hat{t}_c(\underline{p}) := \sup \left\{ t \geqslant 0 \ : \ \mathbb{E}\left(\left| V(G^{\infty}_{\underline{p}}(t)) \right| \right) < \infty \right\} \tag{3}$$

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- There exists d_0 such that for $d \geqslant d_0$, the random d-process has significantly **more edges** when the giant component emerges than the Erdős–Rényi graph process (where there are no degree constraints)
- This result answers a non-trivial open problem of Warnke and Wormald for large values of d

Fine properties of $\hat{t}_c$

- We prove an asymptotic expansion of $\hat{t}_c$
- If each degree constraint is d (**random *d*-process**), then $\hat{t}_c = \hat{t}_c(d)$ satisfies

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Open question: Does this hold for $d = 3, 4, \ldots, d_0 - 1$?

Fine properties of $\hat{t}_c$

Nearly 2-regular case:

Fine properties of $\hat{t}_c$

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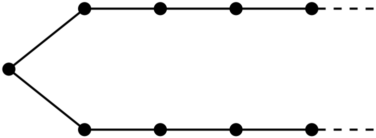
- Random 2-process: $\hat{t}_c = \infty$



Local limit in the 2-regular case

Fine properties of $\hat{t}_c$

Nearly 2-regular case:

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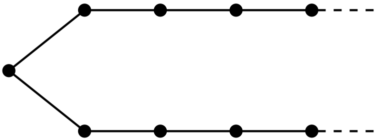

Local limit in the 2-regular case

- If the i.i.d. degree constraints are equal to 2 with probability $1-\varepsilon$, then $\hat{t}_c$ converges to ∞ as $\varepsilon \to 0$

Fine properties of $\hat{t}_c$

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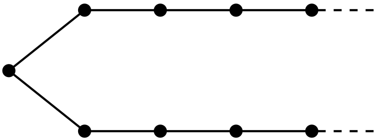

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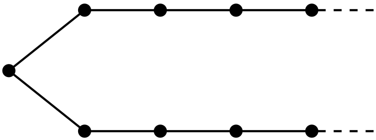

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$$\hat{t}_c(\underline{p}) \sim \frac{1}{\sum_{k=3}^{\Delta} k(k-2)p_k} = \frac{1}{\mathbb{E}[D(D-2)]}, \tag{5}$$

where Δ is an upper bound on $\underline{p}$ and $D \sim \underline{p}$

Thank you for your attention!

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https://arxiv.org/pdf/2409.11747