Asymptotics of Random Tilings of the Aztec Diamond

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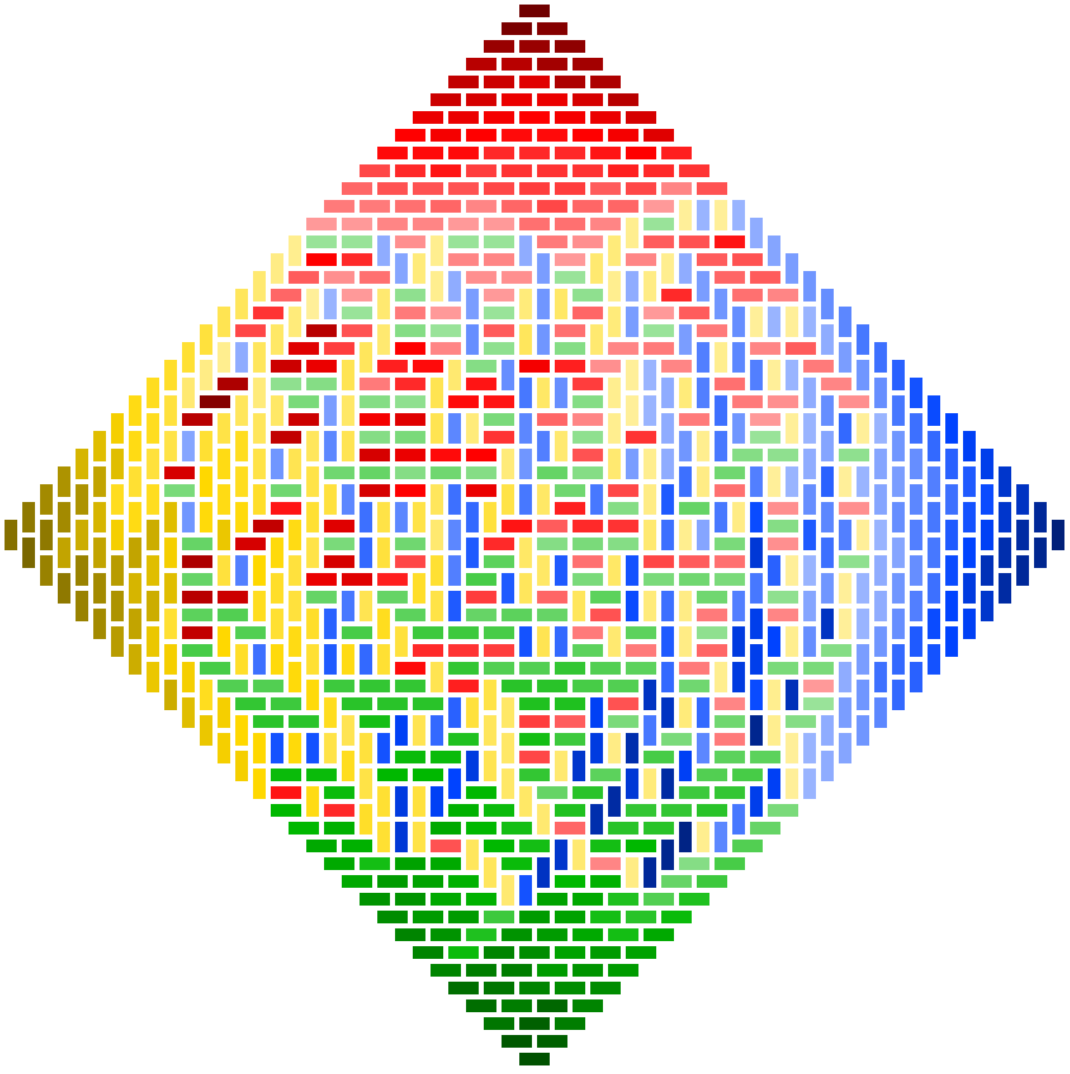
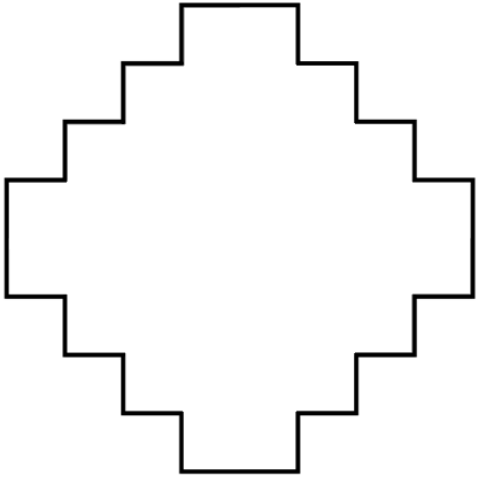
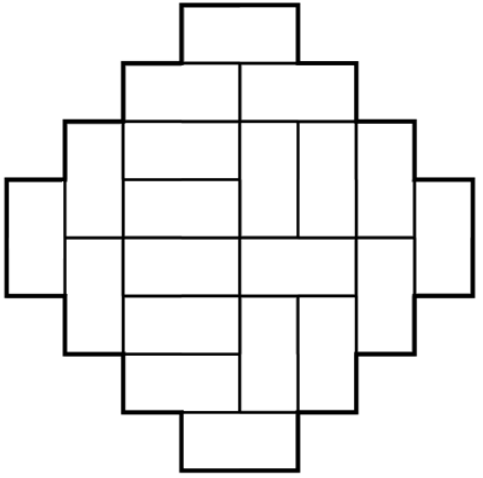
Introduction



The Aztec diamond domain and a domino tiling

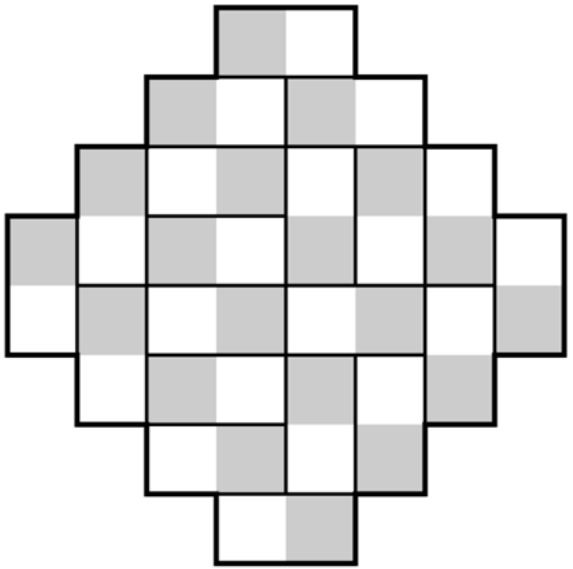


Aztec diamond domain of order 4

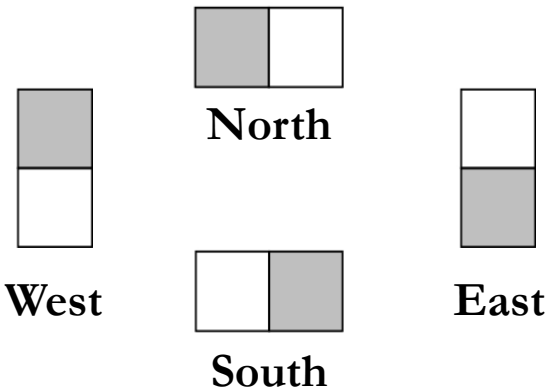


Sample domino tiling

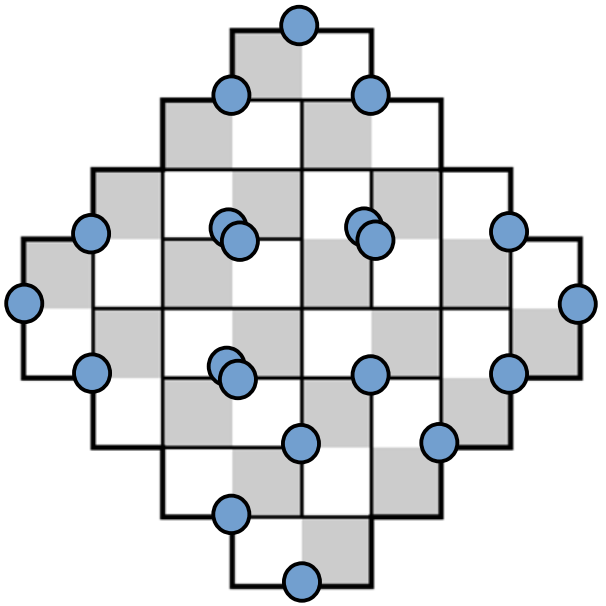
The four types of dominoes



Checkerboard coloring



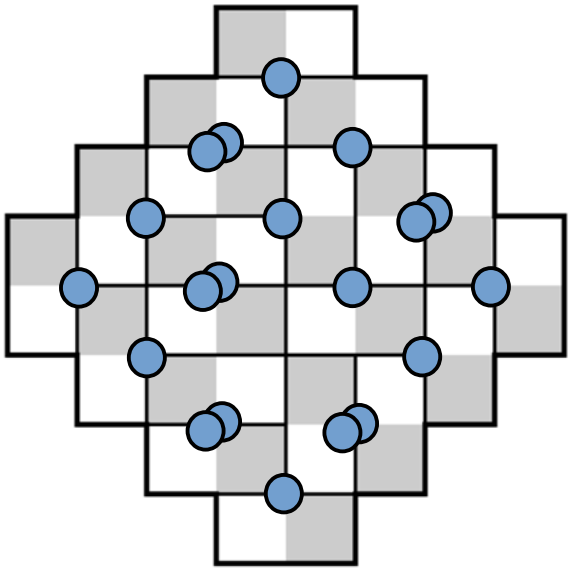
Pairs of alternating sign matrices (ASMs)



$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

Elkies et. al. – Alternating-Sign Matrices and Domino Tilings I, 1992

Pairs of alternating sign matrices (ASMs)



0	0	1	0
1	0	0	0
0	1	–1	1
0	0	1	0

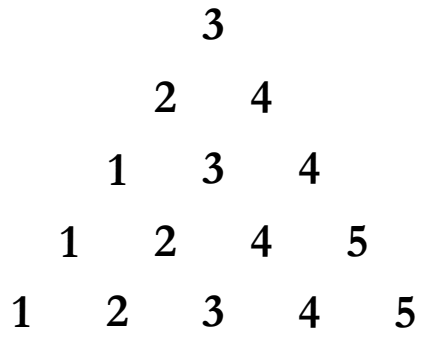
B

Elkies et. al. – Alternating-Sign Matrices and Domino Tilings I, 1992

ASMs and monotone triangles

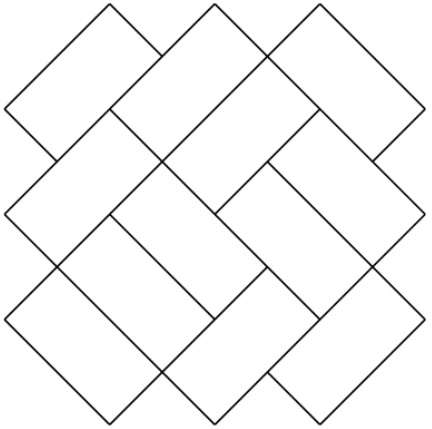
0	0	1	0	0
0	1	–1	1	0
1	–1	1	0	0
0	1	–1	0	1
0	0	1	0	0

0	0	1	0	0
0	1	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

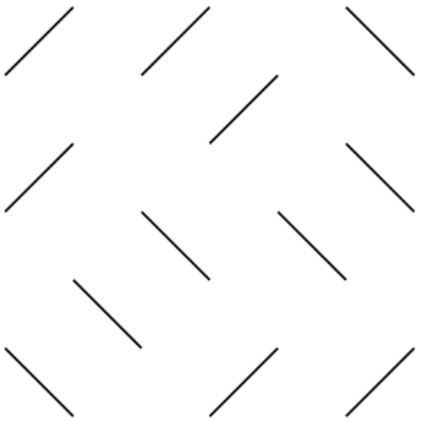


Elkies et. al. – Alternating-Sign Matrices and Domino Tilings I, 1992

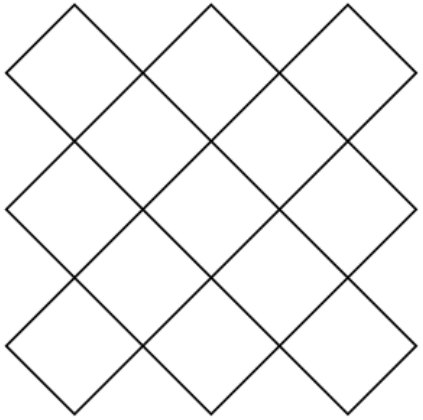
Dimer configurations = perfect matchings of the Aztec diamond graph



Sample domino tiling



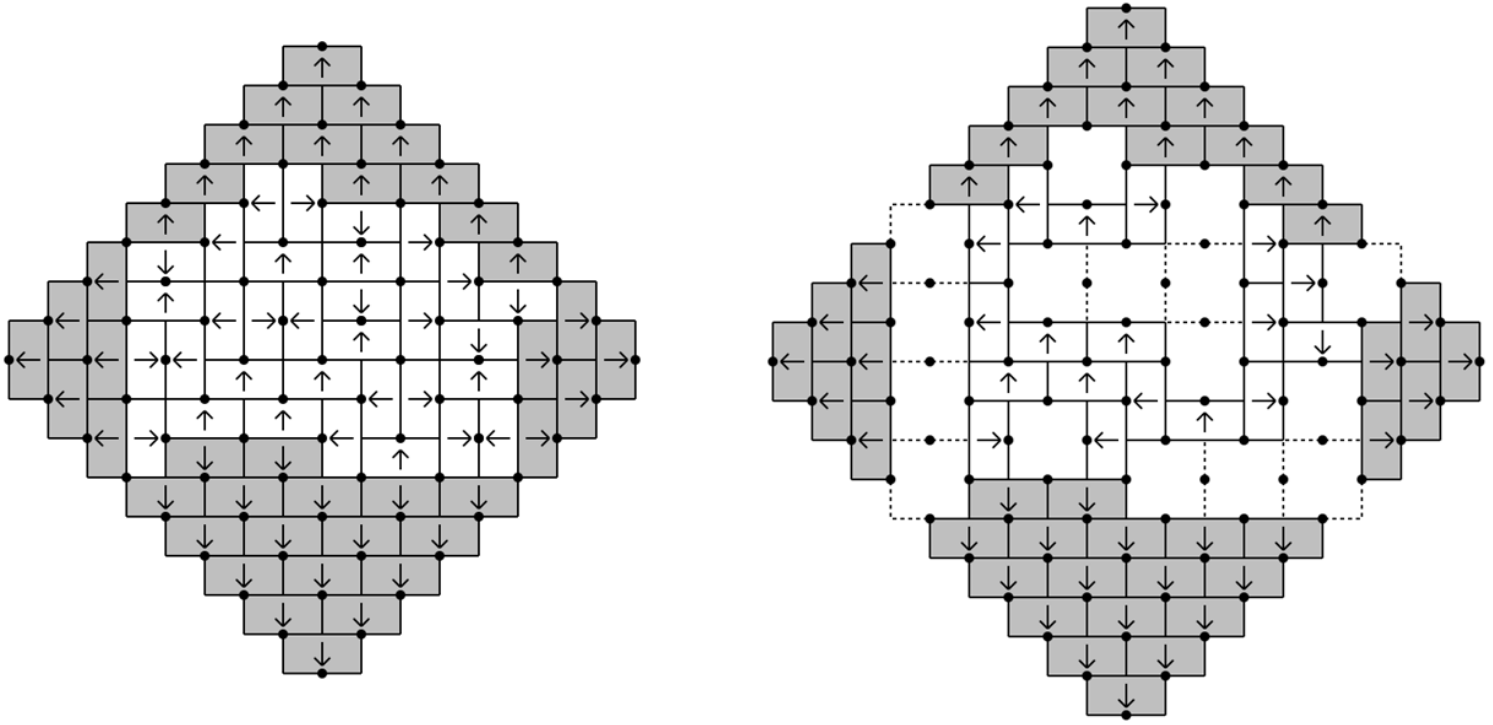
Dimer configuration
Perfect matching



Aztec diamond graph

James Propp – Generalized Domino-Shuffling, 2001

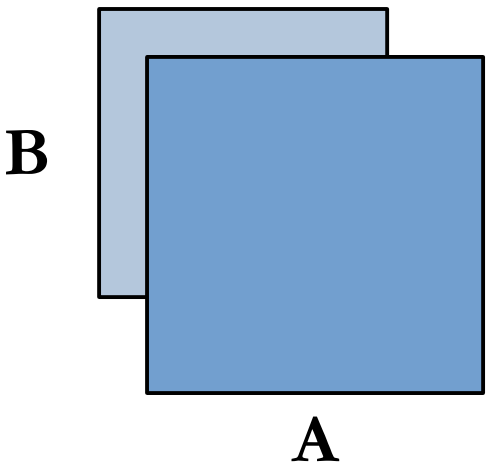
The shuffling algorithm

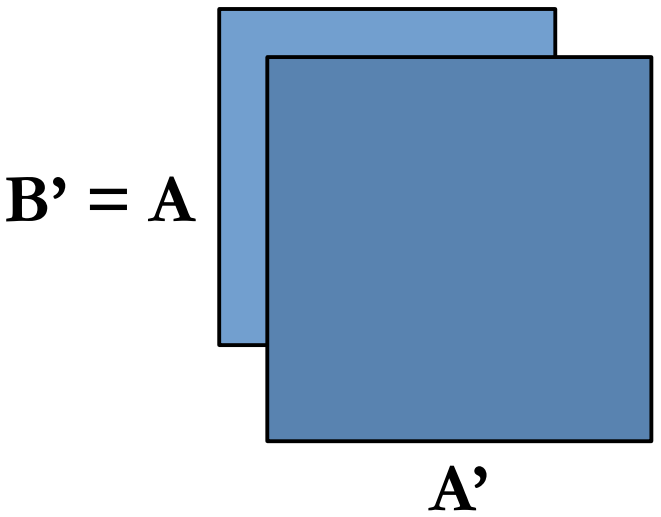


Destruction, sliding, creation

Jockusch, Propp, Shor – Random domino tilings and the arctic circle theorem (1995)

The shuffling algorithm



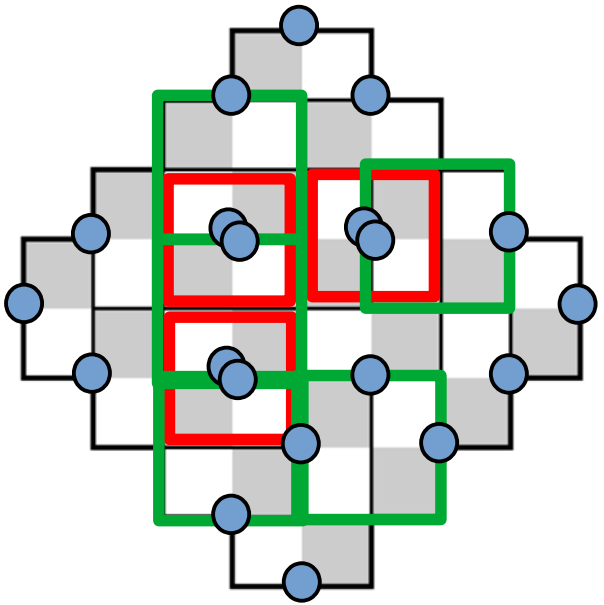


$T = (A, B)$

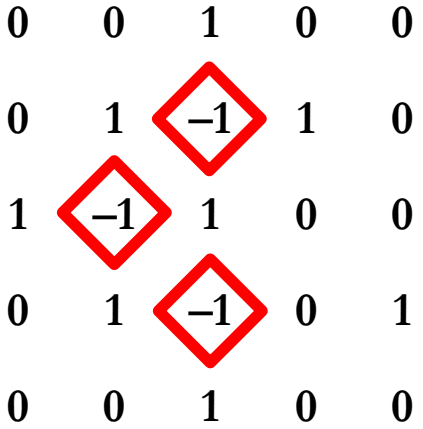
$T' = (A', B')$

Jockusch, Propp, Shor – Random domino tilings and the arctic circle theorem (1995)

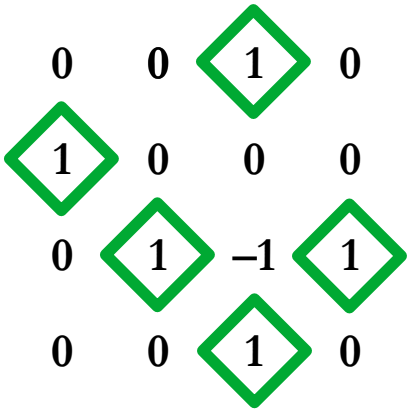
Preserving the information lost during destruction



T’ = (A’, B’)

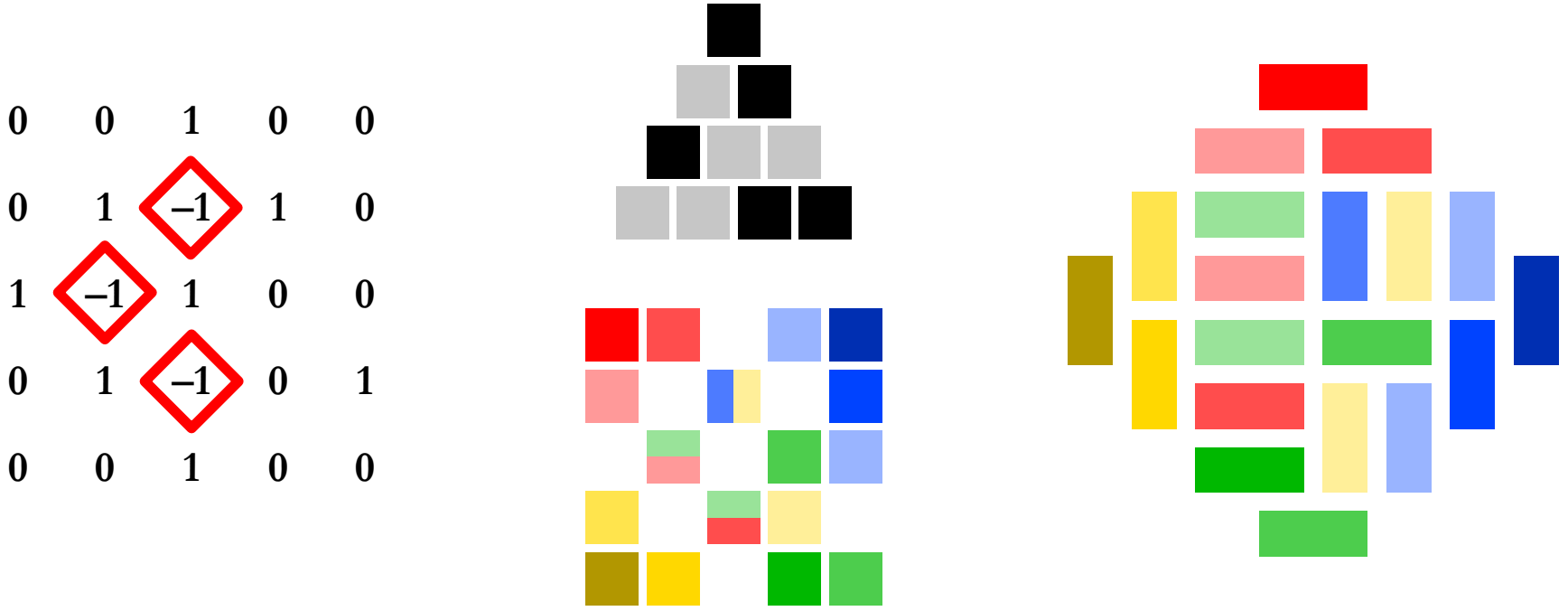


A’



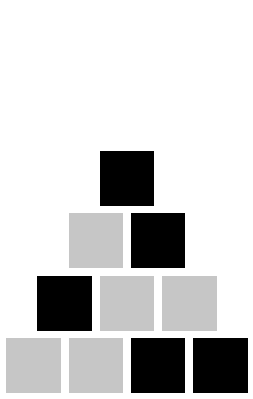
B’ = A

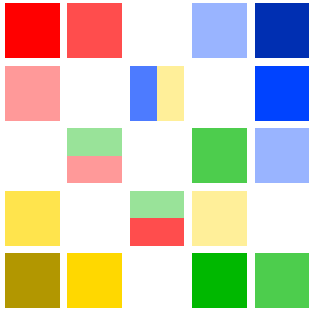
Bijection between tilings and triangles of bits

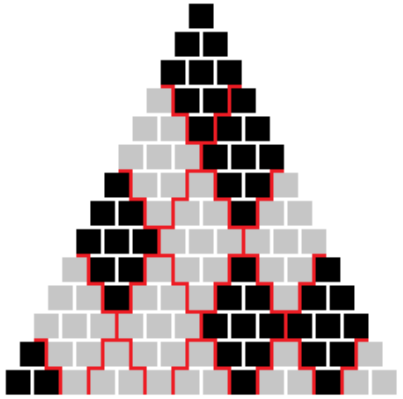


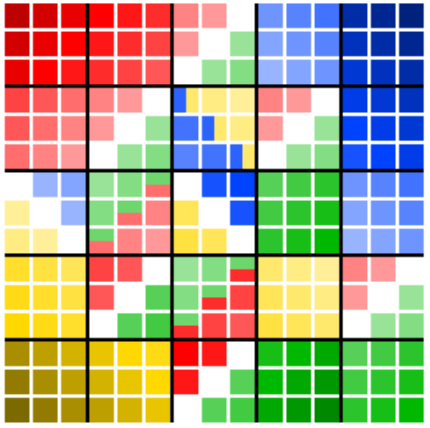
Bosio, van Leeuwen – A Bijection Proving the Aztec Diamond Theorem by Combining Lattice Paths (2012)

Scaling

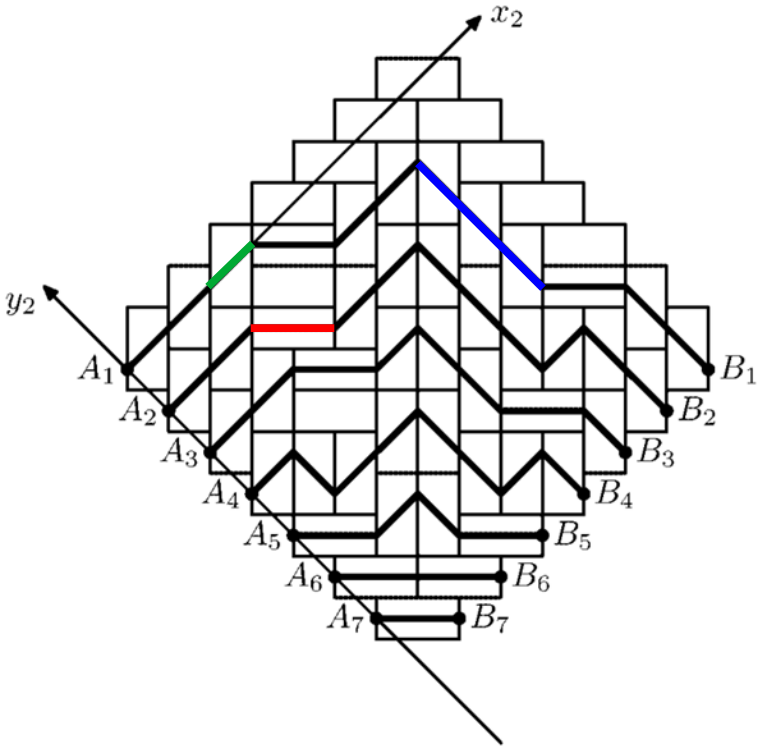


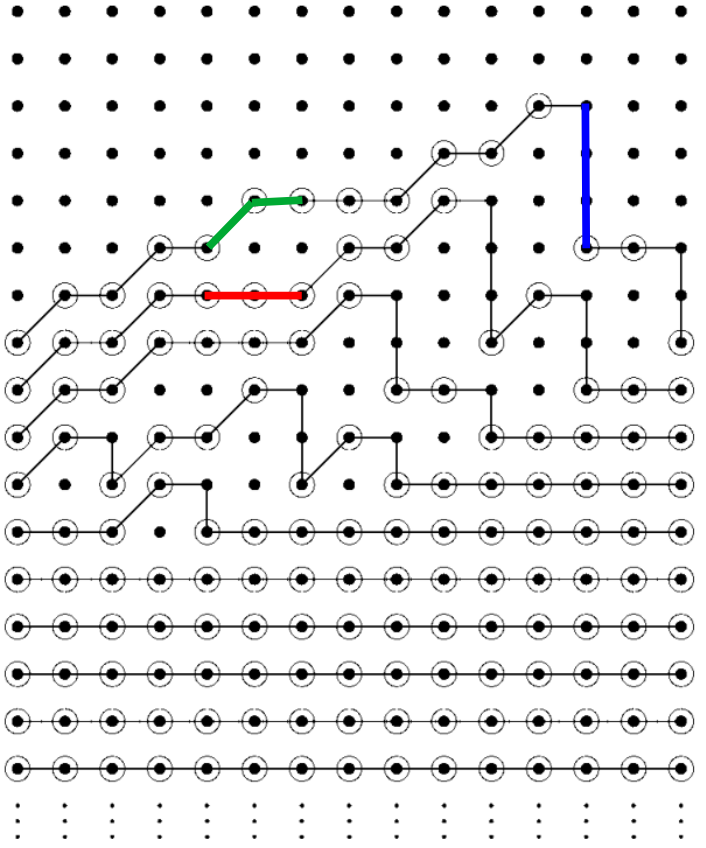






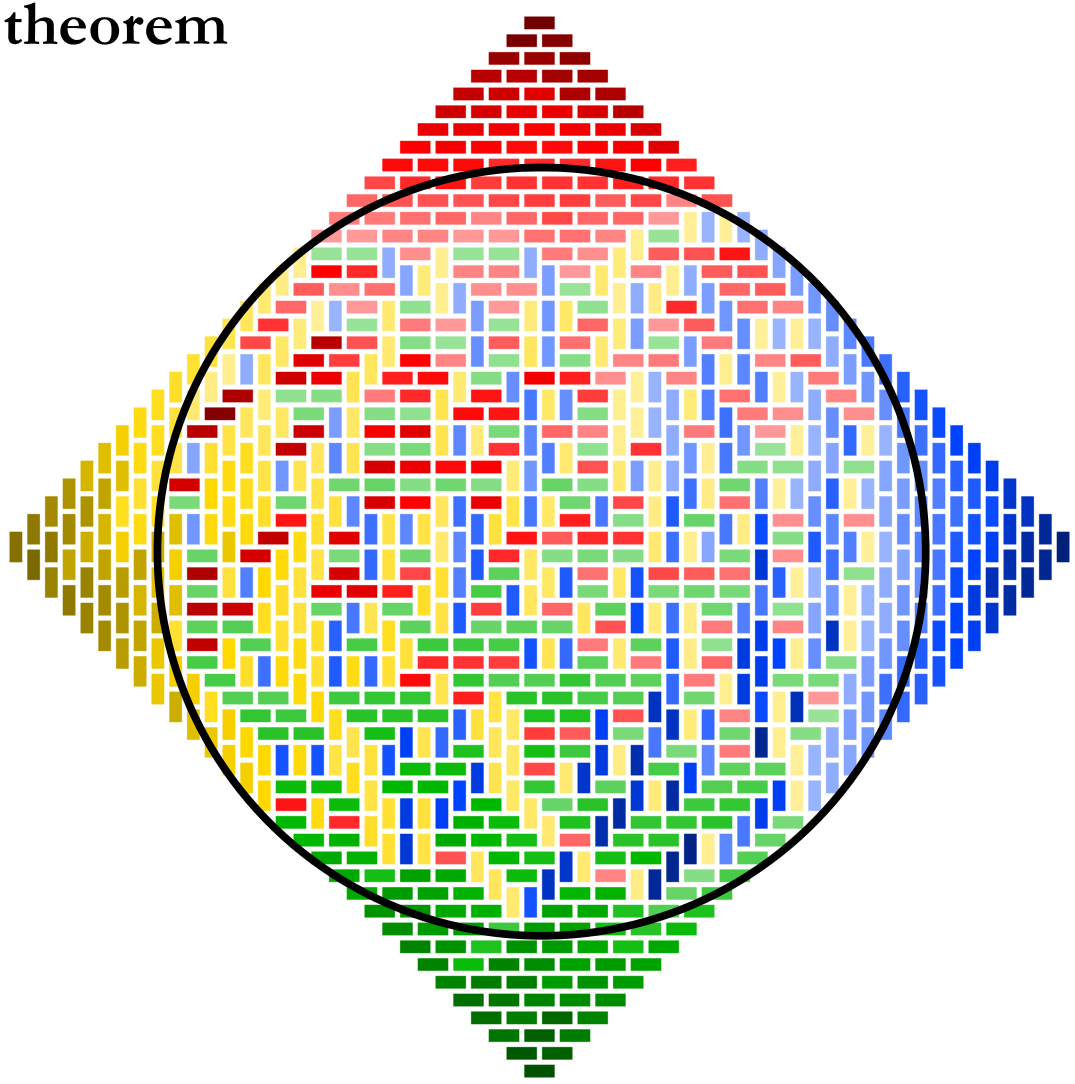
Tilings and determinantal point processes





Johansson – The Arctic Circle Boundary and the Airy Process, 2005

The arctic circle theorem



Closing examples