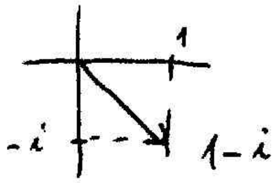
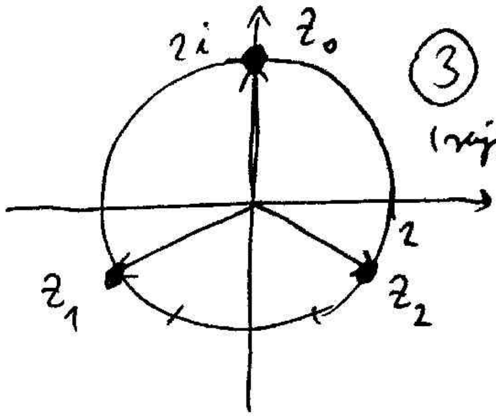
Szá Mód Tin-1. | 1. ZH. | -1- | (2018. 11. 25.

1, $(1-i)^8 = \left(\sqrt{2}\; e^{-i\frac{\pi}{4}}\right)^8 = 2^4 \cdot \underbrace{e^{i\frac{8\pi}{4}}}_{1} = 16$ (2)



$z^3 = \frac{16}{2i} = -8i = 8 \cdot e^{\frac{3\pi}{2} i}$

$z_k = \sqrt[3]{8} \cdot e^{i\left(\frac{\pi}{2} + k\frac{2\pi}{3}\right)}$; $k = 0, 1, 2$ (2)



(3) (rajz)

$z_0 = 2 \cdot e^{i\pi/2} = \underline{\underline{2i}}$ (1)

$z_1 = 2 \cdot e^{i\frac{7\pi}{6}} = \underline{\underline{-\sqrt{3} - i}}$ (1)

$z_2 = 2\, e^{i\frac{11\pi}{6}} = \underline{\underline{+\sqrt{3} - i}}$ (1)

2, $p(z) = z^2 + 1 = (z+i)(z-i)$ (1)

$q(z) = z^2 + iz + 2 = (z-i)(z+2i)$ (3)

$z_{12} = \frac{-i \pm \sqrt{-1-8}}{2} = \frac{-i \pm 3i}{2} = \begin{cases} -2i \\ +i \end{cases}$

a, $m(z) = \underbrace{(z+i)(z-i)}_{z^2+1}(z+2i) = \underline{\underline{z^3 + 2iz^2 + z + 2i}}$ (3)

b, $d(z) = \underline{\underline{z - i}}$ (3)

3, $\underline{n} = \underline{a} \times \underline{b} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \times \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4-3 \\ 0-2 \\ -3-0 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$ (3) ; $|\underline{n}| = \sqrt{14}$ (2)

$\underline{\underline{P}} = \underline{\underline{I}} - \frac{\underline{n} \circ \underline{n}}{|\underline{n}|^2}$ (3) $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \frac{1}{14}\begin{bmatrix} 1 & -2 & -3 \\ -2 & 4 & 6 \\ -3 & 6 & 9 \end{bmatrix} = \underline{\underline{\frac{1}{14}\begin{bmatrix} 13 & 2 & 3 \\ 2 & 10 & -6 \\ 3 & -6 & 5 \end{bmatrix}}}$ (2)

4, $\underline{a} := \vec{BA} = \vec{A} - \vec{B} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ (2) ; l irányvektora: $\underline{b} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ (2)

A keresett sík normálvektora: $\underline{n} = \underline{a} \times \underline{b} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -6 \end{bmatrix}$ (3)

A sík egyenlete: $\underline{\underline{3x + 2y - 6z}} = 3\cdot 1 + 2\cdot 0 - 6\cdot 2 = \underline{\underline{-9}}$ (3)

— ∘ —

5, a, indexesen!

$$(\underline{a} \times \underline{b}) \cdot (\underline{a} \times \underline{b}) = \varepsilon_{ijk}\, a_j b_k \cdot \varepsilon_{ilm}\, a_l b_m \overset{(2)}{=} (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl})\, a_j a_l b_k b_m =$$

$$= a_j a_j b_k b_k - a_j b_j a_k b_k = \underline{\underline{|\underline{a}|^2 \cdot |\underline{b}|^2 - (\underline{a}\cdot\underline{b})^2}} \quad (3)$$

b, kifejtési tétellel:

$$(\underline{a} \times \underline{b}) \cdot (\underline{a} \times \underline{b}) = (\underline{a}, \underline{b}, \underline{a} \times \underline{b}) = \underline{a} \cdot \underbrace{(\underline{b} \times (\underline{a} \times \underline{b}))}_{\underline{a}\cdot|\underline{b}|^2 - \underline{b}\cdot(\underline{a}\cdot\underline{b})} = \underline{\underline{|\underline{a}|^2 \cdot |\underline{b}|^2 - (\underline{a}\cdot\underline{b})^2}} \quad (5)$$

— ∘ —

6, Linearitás: $\forall\, p_1, p_2 \in W$, $\alpha \in \mathbb{R}$ esetén:

$$(A(\alpha p_1 + p_2))(x) = (\alpha p_1 + p_2)(f(x)) = \alpha p_1(f(x)) + p_2(f(x)) =$$
$$= \alpha (Ap_1)(x) + (Ap_2)(x) = (\alpha A p_1 + A p_2)(x) \quad \forall x\text{-re} \quad (3)$$

$\Rightarrow A(\alpha p_1 + p_2) = \alpha A p_1 + A p_2$

Legyen $q_0(x) = 1$, $q_1(x) = x$, $q_2(x) = x^2$.

$(A q_0)(x) = q_0(f(x)) = 1 = q_0(x) \quad \Rightarrow A q_0 = q_0$ (1)

$(A q_1)(x) = q_1(f(x)) = 2x - 1 = (2q_1 - q_0)(x) \Rightarrow A q_1 = 2q_1 - q_0$ (2)

$(A q_2)(x) = q_2(f(x)) = (2x-1)^2 = 4x^2 - 4x + 1 = (4q_2 - 4q_1 + q_0)\, x$

$\Rightarrow A q_2 = 4q_2 - 4q_1 + q_0$ (2)

Tehát

$$[A] = \begin{bmatrix} 1 & -1 & +1 \\ 0 & 2 & -4 \\ 0 & 0 & +4 \end{bmatrix} \quad (2)$$