

1, a,  $\dim V = d$  este  $V^{\wedge d}$  egdimensionis, și

$A^{\wedge d} \in \text{Lin}(V^{\wedge d})$  cu valoare minimală zero. De a  $\det A_{\min} = \det A$ .  $\quad \textcircled{3}$

b, At  $\gamma: l^{\infty} \rightarrow (l^1)^*$ ,  $x \mapsto \gamma_x$ ;  $\forall y \in l^1: \gamma_x(y) = \sum_{k=0}^{\infty} x_k y_k$   $\quad \textcircled{4}$   
liniară liniară, izometriku bijektivă.

c,  $(Q, d)$ , astfel  $d(x, y) = |x - y|$ .  $\quad \textcircled{3}$

$$2, (A \vee A)(e \vee e) = Ae \vee Ae = (e + of) \vee (e + of) = e \vee e$$

$$(A \vee A)(e \vee f) = Ae \vee Af = e \vee (2e - f) = 2e \vee e - e \vee f$$

$$(A \vee A)(f \vee f) = Af \vee Af = (2e - f) \vee (2e - f) = 4e \vee e - 4e \vee f + f \vee f$$

$$\Rightarrow [A \vee A] = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

$\textcircled{10}$

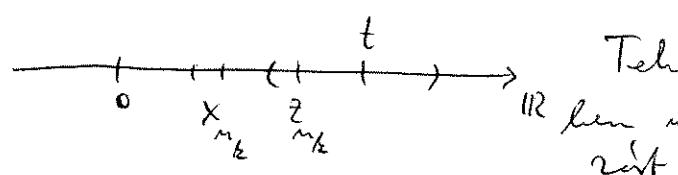
3, a, Lăsă  $x = (x_n)_{n \in \mathbb{N}}$ ,  $y = (y_n) \in C_0$ , și  $\alpha, \beta \in \mathbb{R}$ . Elă

$$\lim_{n \rightarrow \infty} (\alpha x + \beta y)_n = \lim_{n \rightarrow \infty} (\alpha x_n + \beta y_n) = \underbrace{\alpha \lim_{n \rightarrow \infty} x_n}_0 + \underbrace{\beta \lim_{n \rightarrow \infty} y_n}_0 = 0 \quad \textcircled{4}$$

b, Lăsă  $z \in l^{\infty} \setminus C_0$ , tehnă nu nullă convergență nu are.

$\Rightarrow z$  nu este egala cu o terodă punctă.  $\varepsilon := |t|/3$ , și lăsă  
 $x = (x_n) \in B_{\varepsilon}(z)$ . Tău  $x = (x_n)$  nu este terodă punctă, tehnă  $\exists (x_{n_k})$   
nuconvergentă, nuță  $|x_{n_k} - z| < \varepsilon$ . Elă arăta

$$|x_{n_k}| = |x_{n_k} - z + z - t + t| \geq |t| - \underbrace{|x_{n_k} - z|}_{< \varepsilon = |t|/3} - \underbrace{|z - t|}_{< \varepsilon = |t|/3} \geq |t| - 2|t|/3 = |t|/3 > 0$$



Tehnă  $x$  nu este nuconvergentă. Tehnă  $B_{\varepsilon}(z)$  nu este terodă punctă.  
nuță  $\Rightarrow l^{\infty} \setminus C_0$  nu este  $C_0$ .

$$\begin{aligned}
 4, a, \|x \otimes y\|_2 &= \sqrt{\sum_{i=1}^m \sum_{j=1}^n |x_i y_j|^2} = \sqrt{\left(\sum_{i=1}^m |x_i|^2\right) \cdot \left(\sum_{j=1}^n |y_j|^2\right)} = \\
 &= \sqrt{\sum_{i=1}^m |x_i|^2} \cdot \sqrt{\sum_{j=1}^n |y_j|^2} = \|x\|_2 \cdot \|y\|_2 \quad \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 b, \phi(x+h) &= (x+h) V(x+h) = \underbrace{x V x}_{\phi(x)} + \underbrace{X V h}_{2 X V h} + \underbrace{h V x}_{h \text{-ban lineär}} + \underbrace{h V h}_{\text{stetig}} \quad \checkmark
 \end{aligned} \tag{5}$$

$$\begin{aligned}
 d\phi[x]h &= 2XVh \\
 (\text{Naheln, da alle gängige}) \quad \frac{\|hVh\|_2}{\|h\|_2} &= \frac{\frac{1}{\sqrt{2}}\|h \otimes h\|_2}{\|h\|_2} = \frac{1}{\sqrt{2}} \frac{\|h\|_2^2}{\|h\|_2} = \frac{1}{\sqrt{2}} \|h\|_2 \xrightarrow{\|h\|_2 \rightarrow 0} 0
 \end{aligned}$$

(Vejts dimension weiteres bilden ist norm schwächer.)

5, Felsö" basis:

$$|\phi(f)| = \left| \int_0^1 f(t) dt + 2f'(0) \right| \leq \int_0^1 \|f\|_\infty dt + 2\|f'\|_\infty = \|f\|_\infty + 2\|f'\|_\infty \leq$$

$$\leq 2\|f\|_\infty + 2\|f'\|_\infty = 2\|f\|, \text{ teilt } \underline{\|\phi\| \leq 2} \quad \text{(Gesucht, da} \quad \text{Felsö" basis: 2p)}$$

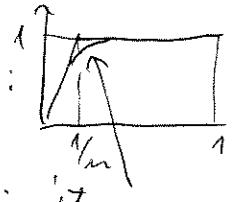
Alm basis:

$$\text{denn } f_m(x) = \sin(2m\pi x)$$

$$\|f_m\|_\infty = 1, \|f'_m\|_\infty = \|\sin(2m\pi x)\|_\infty = 2m\pi \Rightarrow \|f_m\| = 1 + 2m\pi$$

$$|\phi(f_m)| = \left| \int_0^1 2\sin(2m\pi x) dx + 4m\pi \right| = 4m\pi; \frac{|\phi(f_m)|}{\|f_m\|} = \frac{4m\pi}{1+2m\pi} \xrightarrow{m \rightarrow \infty} 2$$

$$\text{Tebt } \underline{\|\phi\| \geq \frac{4m\pi}{1+2m\pi}} \quad \forall m \in \mathbb{N} \Rightarrow \underline{\|\phi\| \geq 2} \Rightarrow \underline{\|\phi\| = 2}$$

Alm basispl a  $g_m(x)$ :  Die period. ist  $\frac{1}{m}$ .

(Gesucht positiv alm basis: 2p)