

1, a, Banach-Steinhaus Titel:

W Ha X Banach-Gr., is a $\{T_i\}_{i \in I} \subset B(X)$ operator - contd. pointwise
kontinuität, also normabn is folglt. 3 p

b, Kyli Lépési titel:

Ha X, Y Banach-Gr., $A \in B(X, Y)$ surjektiv, akkor A spst. 3

c, Riesz-Fréchet representation titel: Legyen H Hilbert-Gr. Első a

$\gamma: H \rightarrow H^*$, $x \mapsto \gamma_x$; $\forall \gamma \in H: \gamma_x(\gamma) := \langle x, \gamma \rangle$

Lépési normával körülött - lineáris bijekcio. 4

$$\left. \begin{aligned} 2, a, \|A(\gamma)\| &= \|\|\gamma\|^2 \cdot z\| = \|\gamma\|^2 \cdot \|z\| \Rightarrow \|A\| \geq \|\gamma\| \cdot \|z\| \\ \|A \times\| &= |(\gamma, x)| \cdot \|z\| \stackrel{\text{Sikom}}{\leq} \|\gamma\| \cdot \|x\| \cdot \|z\| \Rightarrow \|A\| \leq \|\gamma\| \cdot \|z\| \end{aligned} \right\} \|A\| = \|\gamma\| \cdot \|z\|$$
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$$\begin{aligned} b, \langle v, Aw \rangle &= \langle v, (\gamma, w)z \rangle = \underbrace{\langle (\gamma, w)v, z \rangle}_{\langle v, z \rangle} = \\ &= \langle \gamma, w \rangle \langle v, z \rangle = \underbrace{\langle \langle z, v \rangle \gamma, w \rangle}_{\langle v, z \rangle} = \langle A^*v, w \rangle, \end{aligned}$$

tehet $A^*v = \langle z, v \rangle \gamma$ 6

$$c, A = A^*, \text{ ha } \forall x \in H: (\gamma, x)z = \langle z, x \rangle \gamma \Leftrightarrow$$

$$(\Leftrightarrow) \gamma = 0 \vee \gamma \neq 0 \text{ végz } z = \alpha \gamma, \text{ ahol } \alpha \in \mathbb{R}. \quad \text{3}$$

$$\left. \begin{aligned} 3, \forall x \in \ker T, \text{ akkor } Tx = 0 &\Rightarrow T^*Tx = 0 \Rightarrow \ker T \subset \ker(T^*T) \\ \forall x \in \ker(T^*T), \text{ akkor } T^*Tx = 0 &\Rightarrow \langle x, T^*Tx \rangle = 0 \Rightarrow \\ \Rightarrow \langle Tx, Tx \rangle &= \|Tx\|^2 = 0 \Rightarrow Tx = 0 \Rightarrow x \in \ker T \end{aligned} \right\} \ker T = \ker(T^*T) \quad \checkmark$$
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$$6, \quad x = (x_0, x_1, x_2, \dots) \quad \boxed{-1} \quad Ax = (x_0, x_2, x_4, \dots)$$

$$a(\gamma, Ax) = \langle (\gamma_0, \gamma_1, \gamma_2, \dots), (x_0, x_2, x_4, \dots) \rangle =$$

$$\textcircled{5} \quad = \bar{\gamma}_0 \cdot x_0 + \bar{\gamma}_1 \cdot x_2 + \bar{\gamma}_2 \cdot x_4 + \dots = \sum_{k=0}^{\infty} \bar{\gamma}_k x_{2k} =$$

$$= \langle (\gamma_0, 0, \gamma_1, 0, \gamma_2, 0, \dots), (x_0, x_1, x_2, \dots) \rangle,$$

teilt $A^* x = (x_0, 0, x_1, 0, \dots)$, $A^* \mathcal{S}_\ell = \mathcal{S}_{2\ell}$, $[A^* x]_i = \begin{cases} 0, & \text{h.a.p.t.h.} \\ x_{i/2}, & \text{h.a.i.p.s.} \end{cases}$

b) w.o.t.:

$$\langle A^n x, \gamma \rangle = \langle (x_0, x_{2^n}, x_{2 \cdot 2^n}, x_{3 \cdot 2^n}, \dots), (\gamma_0, \gamma_1, \gamma_2, \dots) \rangle =$$

$$= \sum_{k=0}^{\infty} \bar{x}_{k \cdot 2^n} \cdot \gamma_k = \bar{x}_0 \gamma_0 + \sum_{k=1}^{\infty} \bar{x}_{k \cdot 2^n} \gamma_k, \text{ s.t.}$$

$$\left| \sum_{k=1}^{\infty} \bar{x}_{k \cdot 2^n} \gamma_k \right| \leq \sqrt{\sum_{k=1}^{\infty} |x_k|^2} \cdot \|\gamma\| \xrightarrow{n \rightarrow \infty} 0$$

Teilt $A^n \xrightarrow{w.o.t.} P_0$, und $P_0 x = (x_0, 0, 0, 0, \dots)$,

$$P_0 \mathcal{S}_n = \begin{cases} \mathcal{S}_0, & \text{h.a. } n=0 \\ 0, & \text{h.a. } n \geq 1 \end{cases}$$

s.o.t.:

da \exists s.l. $\lim_{n \rightarrow \infty} A^n$, aber auch ist die P_0 . Da

$$(P_0 - A^n) \mathcal{S}_{2^n} = 0 - \mathcal{S}_1, \text{ teilt } \|P_0 - A^n\| \geq 1, \text{ teilt}$$

ein op. top.-dim A^n diverges \Rightarrow normalkonvergenz divergiert.

$$\overline{5, \quad |\phi(x)| \leq \|\phi\| \cdot \|x\| \Rightarrow \|x\| \geq \sup \left\{ \frac{|\phi(x)|}{\|\phi\|} \mid \phi \in X^*, \phi \neq 0 \right\}}$$

6) Analog zu einem Banachraum teilt $\|\phi\| = \sup \{|\phi(x)| \mid x \in X, x \neq 0\}$

$$\exists \phi_x \in X^*, \|\phi_x\| = 1, \|\phi_x(x)\| = \|x\|. \text{ Teilt}$$

$$\sup \left\{ \frac{|\phi(x)|}{\|\phi\|} \mid \phi \in X^*, \phi \neq 0 \right\} \geq \frac{|\phi_x(x)|}{\|\phi_x\|} = \frac{\|x\|}{1} = \|x\| \quad \checkmark$$