1. (4p) **A csoport**

$$\left[\begin{array}{ccc|c} 4 & -1 & 6 & -3 \\ 2 & 1 & 5 & -3 \\ 1 & -3 & 2 & 2 \end{array}\right] \overset{\frac{1}{4}S_1}{\sim} \left[\begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{3}{2} & -\frac{3}{4} \\ 2 & 1 & 5 & -3 \\ 1 & -3 & 2 & 2 \end{array}\right] \underset{S_3-S_1}{\overset{S_2-2S_1}{\sim}}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & \frac{3}{2} & 2 & -\frac{3}{2} \\ 0 & -\frac{11}{4} & \frac{1}{2} & \frac{11}{4} \end{array}\right] \overset{\frac{2}{3}S_2}{\sim} \left[\begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & 1 & \frac{4}{3} & -1 \\ 0 & -\frac{11}{4} & \frac{1}{2} & \frac{11}{4} \end{array}\right] \overset{S_3+\frac{11}{4}S_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & 1 & \frac{4}{3} & -1 \\ 0 & 0 & \frac{23}{3} & 0 \end{array}\right] \overset{\frac{3}{23}S_4}{\sim} \left[\begin{array}{ccc|c} 1 & -\frac{1}{4} & \frac{3}{2} & -\frac{3}{4} \\ 0 & 1 & \frac{4}{3} & -1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$z = 0$$

$$y = -1 - \frac{4}{3}z = -1 - \frac{4}{3}\cdot 0 = -1$$

$$x = -\frac{3}{4} + \frac{1}{4}y - \frac{3}{2}z = -\frac{3}{4} + \frac{1}{4}(-1) - \frac{3}{2}\cdot 0 = -1$$

A megoldás: $x = -1$, $y = -1$, $z = 0$.

2.

a) (2p)

$$\det(A)=\begin{vmatrix} 5 & 3 & 1 \\ 2 & -1 & 1 \\ p & 5 & 0 \end{vmatrix} = p\cdot\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} - 5\cdot\begin{vmatrix} 5 & 1 \\ 2 & 1 \end{vmatrix} =$$

$$= p\cdot(3+1) - 5(5-2) = 4p-15 = 0,$$

amiből $p=\frac{15}{4}$ esetén nem invertálható A.

b) (3p) $4p-15=1 \rightsquigarrow p=4$

$$\mathrm{adj}(A)=\begin{bmatrix} -5 & 4 & 14 \\ 5 & -4 & -13 \\ 4 & -3 & -11 \end{bmatrix}^T = \begin{bmatrix} -5 & 5 & 4 \\ 4 & -4 & -3 \\ 14 & -13 & -11 \end{bmatrix}$$

$$A^{-1} = \underbrace{\frac{1}{\det(A)}}_{1}\cdot \mathrm{adj}(A) = \begin{bmatrix} -5 & 5 & 4 \\ 4 & -4 & -3 \\ 14 & -13 & -11 \end{bmatrix}$$

3.

a) (2p)

$$\left.\begin{aligned} 3t-1 &= 2s+7 \\ -2t+5 &= 4s+5 \\ t-5 &= -3s-6 \end{aligned}\right\} \quad \left.\begin{aligned} 3t-2s &= 8 \\ -2t-4s &= 0 \\ t-3s &= -1 \end{aligned}\right\} \Leftrightarrow$$

$$\left[\begin{array}{cc|c} 3 & -2 & 8 \\ -2 & -4 & 0 \\ 1 & 3 & -1 \end{array}\right] \overset{S_1 \leftrightarrow S_3}{\sim} \left[\begin{array}{cc|c} 1 & 3 & -1 \\ -2 & -4 & 0 \\ 3 & -2 & 8 \end{array}\right] \underset{S_2+2S_1}{\overset{S_3-3S_1}{\sim}}$$

$$\left[\begin{array}{cc|c} 1 & 3 & -1 \\ 0 & 2 & -2 \\ 0 & -11 & 11 \end{array}\right] \overset{\frac{1}{2}S_2}{\sim} \left[\begin{array}{cc|c} 1 & 3 & -1 \\ 0 & 1 & -1 \\ 0 & -11 & 11 \end{array}\right] \overset{S_3+11S_2}{\sim}$$

$$\left[\begin{array}{cc|c} 1 & 3 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right] \Leftrightarrow \begin{aligned} t &= -1-3s = 2 \\ s &= -1 \end{aligned}$$

A metszéspont: $\left\{\begin{aligned} x &= -2+7 = 5 \\ y &= -4+5 = 1 \\ z &= 3-6 = -3 \end{aligned}\right\} M(5,1,-3)$

(b) (2p)

Az irányvektorok:

$\underline{v_e}(3,-2,1)$ és $\underline{v_f}(2,4,-3)$

$$\underline{v_e}\times\underline{v_f}=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -2 & 1 \\ 2 & 4 & -3 \end{vmatrix}=$$

$$=(6-4)\underline{i}-(-9-2)\underline{j}+(12+4)\underline{k}=(2,11,16)$$

(c) A sík normálvektora: $\underline{n}(2,11,16)$.

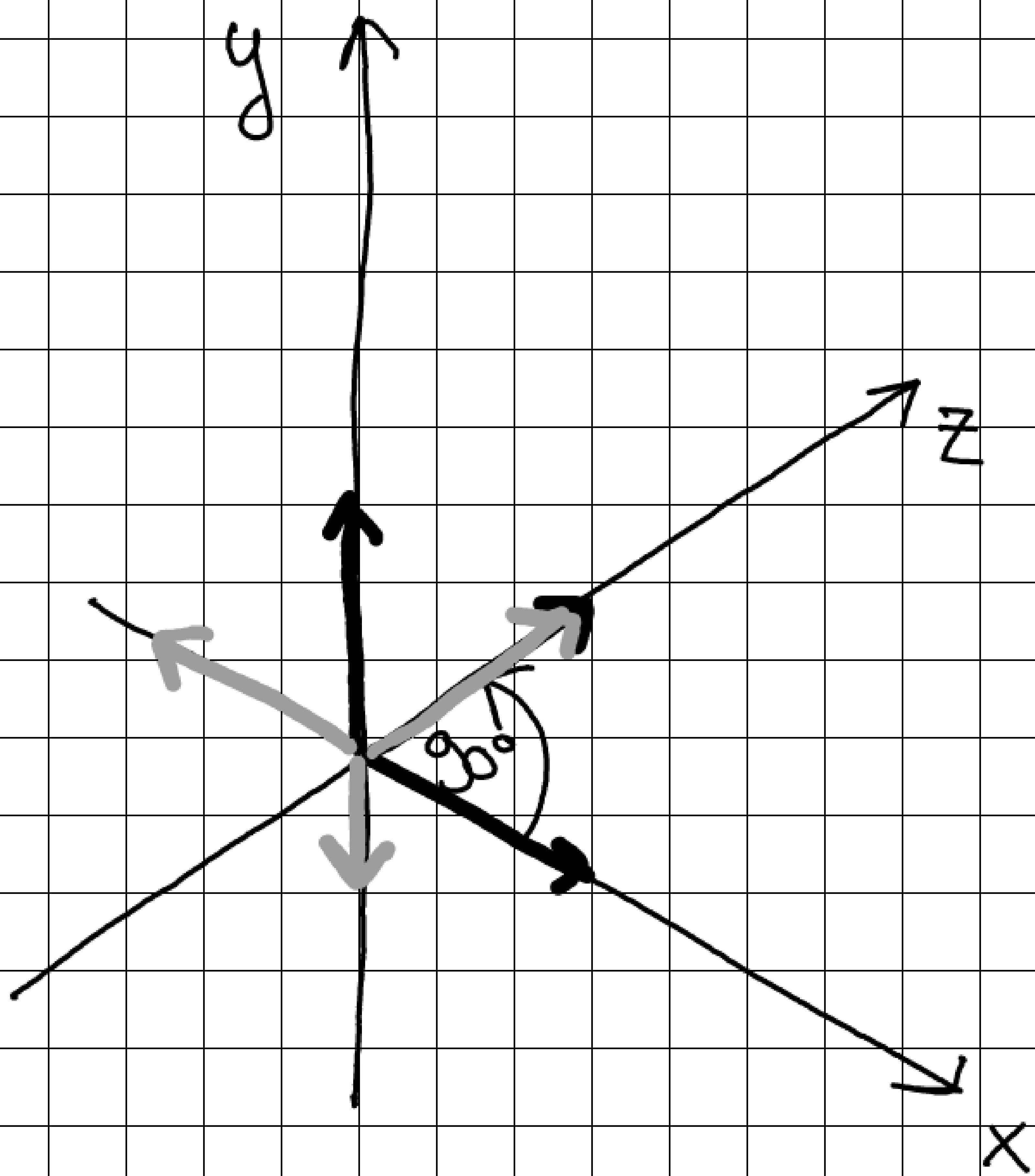
A sík egy pontja: $M(5,1,-3)$.

A sík egyenlete:

$$2x+11y+16z=2\cdot5+11\cdot1+16(-3)$$

$$2x+11y+16z=-27$$

4. (5p)



$$\begin{bmatrix}1\\0\\0\end{bmatrix} \mapsto \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}0\\1\\0\end{bmatrix} \mapsto \begin{bmatrix}0\\-\frac{1}{2}\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} \mapsto \begin{bmatrix}-1\\0\\0\end{bmatrix}$$

$$A = \begin{bmatrix}0 & 0 & -1\\ 0 & -\frac{1}{2} & 0\\ 1 & 0 & 0\end{bmatrix}$$

$$\begin{array}{cc} & \begin{bmatrix}3\\-2\\5\end{bmatrix} \\ \begin{bmatrix}0 & 0 & -1\\ 0 & -\frac{1}{2} & 0\\ 1 & 0 & 0\end{bmatrix} & \begin{bmatrix}-3\\1\\5\end{bmatrix}\end{array}$$

Az $\underline{u}$ vektor képe

$$A\underline{u} = \begin{bmatrix}-3\\1\\5\end{bmatrix}$$

1. (4p) B csoport

$$\left[\begin{array}{ccc|c} 2 & 5 & -1 & -3 \\ 1 & -2 & 3 & 2 \\ 4 & 6 & 1 & -3 \end{array}\right] \overset{\frac{1}{2}S_1}{\sim} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} \\ 1 & -2 & 3 & 2 \\ 4 & 6 & 1 & -3 \end{array}\right] \underset{S_3-4S_1}{\overset{S_2-S_1}{\sim}}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} \\ 0 & -\frac{9}{2} & \frac{7}{2} & \frac{7}{2} \\ 0 & -4 & 3 & 3 \end{array}\right] \overset{-\frac{2}{9}\cdot S_2}{\sim} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & -\frac{7}{9} & -\frac{7}{9} \\ 0 & -4 & 3 & 3 \end{array}\right] \overset{S_3+4S_2}{\sim}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & -\frac{7}{9} & -\frac{7}{9} \\ 0 & 0 & -\frac{55}{9} & -\frac{55}{9} \end{array}\right] \overset{-\frac{9}{55}S_3}{\sim} \left[\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 1 & -\frac{7}{9} & -\frac{7}{9} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$z = 1$

$y = -\frac{7}{9} + \frac{7}{9}z = -\frac{7}{9} + \frac{7}{9} = 0$

$x = -\frac{3}{2} - \frac{5}{2}y + \frac{1}{2}z = -\frac{3}{2} + \frac{1}{2} = -1$

A megoldás: $x = -1$, $y = 0$, $z = 1$.

2.

a) (2p)

$$\det(A)=\begin{vmatrix} 0 & 4 & 5 \\ 1 & 5 & 3 \\ 1 & 2 & p \end{vmatrix} = -4\begin{vmatrix} 1 & 3 \\ 1 & p \end{vmatrix} + 5\cdot\begin{vmatrix} 1 & 5 \\ 1 & 2 \end{vmatrix} =$$

$$= -4(p-3) + 5(2-5) = -3-4p = 0$$

amiből $p=-\frac{3}{4}$ esetén nem invertálható A.

b) (3p) $-3-4p=1 \rightsquigarrow p=-1$

$$\operatorname{adj}(A) = \begin{bmatrix} -11 & 4 & -3 \\ 14 & -5 & 4 \\ -13 & 5 & -4 \end{bmatrix}^T = \begin{bmatrix} -11 & 14 & -13 \\ 4 & -5 & 5 \\ -3 & 4 & -4 \end{bmatrix}$$

$$A^{-1} = \underbrace{\frac{1}{\det(A)}}_{1}\cdot \operatorname{adj}(A) = \begin{bmatrix} -11 & 14 & -13 \\ 4 & -5 & 5 \\ -3 & 4 & -4 \end{bmatrix}$$

3.

a) (2p)

$$\left.\begin{aligned} 3t-2&=s-1\\ -2t+4&=3s-4\\ -4t+6&=-2s+6 \end{aligned}\right\}\quad \left.\begin{aligned} s-3t&=-1\\ 3s+2t&=8\\ -2s+4t&=0 \end{aligned}\right\}\Leftrightarrow$$

$$\left[\begin{array}{cc|c} 1 & -3 & -1\\ 3 & 2 & 8\\ -2 & 4 & 0 \end{array}\right] \underset{S_3+2S_1}{\overset{S_2-3S_1}{\sim}} \left[\begin{array}{cc|c} 1 & -3 & -1\\ 0 & 11 & 11\\ 0 & 1 & 1 \end{array}\right] \overset{\frac{1}{11}S_2}{\sim}$$

$$\left[\begin{array}{cc|c} 1 & -3 & -1\\ 0 & 1 & 1\\ 0 & 1 & 1 \end{array}\right] \overset{S_3-S_2}{\sim} \left[\begin{array}{cc|c} 1 & -3 & -1\\ 0 & 1 & 1\\ 0 & 0 & 0 \end{array}\right] \Leftrightarrow$$

$$\Leftrightarrow \left.\begin{aligned} s-3t&=-1\\ t&=1 \end{aligned}\right\}\quad \begin{aligned} s&=-1+3t=-1+3=2\\ t&=1 \end{aligned}$$

A metszéspont: $\left\{\begin{aligned} x&=2-1=1\\ y&=6-4=2\\ z&=-4+6=2 \end{aligned}\right\} M(1,2,2)$

(b) (2p)

Az irányvektorok:

$\underline{v_e}(3,-2,-4)$ és $\underline{v_f}(1,3,-2)$

$$\underline{v_e} \times \underline{v_f} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -2 & -4 \\ 1 & 3 & -2 \end{vmatrix} =$$

$$= (4+12)\underline{i} - (-6+4)\underline{j} + (9+2)\underline{k} = (16,2,11)$$

(c) A sík normálvektora: $\underline{n}(16,2,11)$.

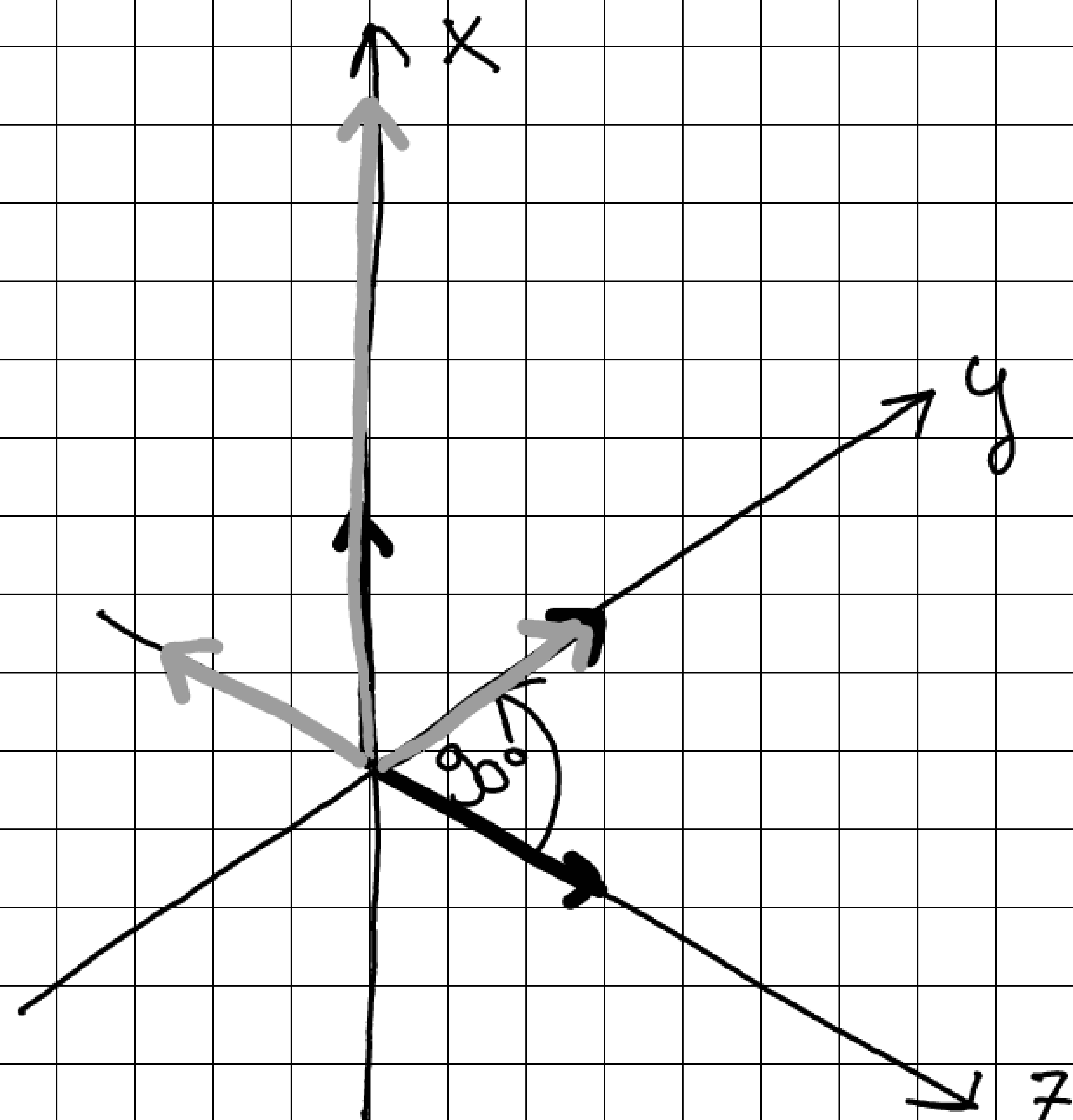
A sík egy pontja: $M(1,2,2)$.

A sík egyenlete:

$$16x + 2y + 11z = 1 \cdot 16 + 2 \cdot 2 + 2 \cdot 11$$

$$16x + 2y + 11z = 42$$

4. (5p)



$$\begin{bmatrix}1\\0\\0\end{bmatrix} \mapsto \begin{bmatrix}3\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0\\1\\0\end{bmatrix} \mapsto \begin{bmatrix}0\\0\\-1\end{bmatrix}$$

$$\begin{bmatrix}0\\0\\1\end{bmatrix} \mapsto \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$$A = \begin{bmatrix}3 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix}$$

$$\begin{array}{cc} & \begin{bmatrix}4\\1\\-2\end{bmatrix} \\ \begin{bmatrix}3 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix} & \begin{bmatrix}12\\-2\\-1\end{bmatrix}\end{array}$$

Az $\underline{u}$ vektor képe

$$A\underline{u} = \begin{bmatrix}12\\-2\\-1\end{bmatrix}$$