

Probability Theory 2

Kolmogorov's 0-1 Law

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- **Tail σ -algebra**

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $(\mathcal{B}_j)_{j=1}^{\infty}$ a sequence of sub- σ -algebras. Let

$$\mathcal{B}_k^n = \sigma \left(\bigcup_{j=k}^n \mathcal{B}_j \right).$$

Then the tail σ -algebra

$$\mathcal{T} = \bigcap_{k=1}^{\infty} \mathcal{B}_k^{\infty}.$$

- **Kolmogorov's 0-1 Law**

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space and $(\mathcal{B}_j)_{j=1}^{\infty}$ a sequence of independent sub- σ -algebras. Then for any $A \in \mathcal{T}$

$$\mathbb{P}(A) = 0 \quad \text{or} \quad \mathbb{P}(A) = 1.$$