

Probability Theory 2

Weak convergence

13.05.2025.

- **Weak convergence**

Let X_n and X be random variables with cumulative distribution functions F_n and F . We say X_n converges to X weakly (or in distribution) if for any continuity point x of F

$$\lim_{n \rightarrow \infty} F_n(x) = F(x).$$

Notations: $X_n \xrightarrow{d} X$, $X_n \Rightarrow X$ or $F_n \Rightarrow F$.

- **Equivalent definition of weak convergence**

The following are equivalent.

(i) $X_n \xrightarrow{d} X$

(ii) for every continuous and bounded $f : \mathbb{R} \rightarrow \mathbb{R}$ $\mathbb{E}(f(X_n)) \rightarrow \mathbb{E}(f(X))$

- **Relationship with other convergences**

Let X_n, X be defined on the same probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Then

$$X_n \xrightarrow{\mathbb{P}} X \quad \text{implies} \quad X_n \xrightarrow{d} X.$$