2nd Midterm

Working time: 45 minutes

Ex1 Let $X_2, X_3, \ldots$ be independent random variables such that X_n has distribution $\mathrm{Exp}(\log(n))$.

(a) (7 points) Show that

$$X_n \xrightarrow{\mathbb{P}} 0 \qquad \text{and} \qquad X_n \xrightarrow{L^2} 0 \qquad \text{as } n \to \infty.$$

(b) (13 points) Show that

$$\mathbb{P}(\limsup_{n\to\infty} X_n = 1) = 1.$$

(Hint: For which values of $c > 0$ do the events $\{X_n > c\}$ happen infinitely often?)

Ex2 (10 points) Let X_1, X_2 be i.i.d. random variables with distribution $\mathrm{Uni}(0, 1)$, and let $Y = \max\{X_1, X_2\}$. Find the characteristic function of Y.