A) hétfő

1)

$$\left(\sqrt{19+8\sqrt{3}}-\sqrt{3}\right)+\sqrt[3]{7^{\log_{\sqrt{7}}2-\log_{49}\frac{1}{4}}}=$$

$$=\left[\sqrt{4^2+2\cdot 4\cdot\sqrt{3}+(\sqrt{3})^2}-\sqrt{3}\right]+\sqrt[3]{\frac{\left[(\sqrt{7})^2\right]^{\log_{\sqrt{7}}2}}{\left(49^{\frac{1}{2}}\right)^{\log_{49}\frac{1}{4}}}}=$$

$$=\left[\sqrt{(4+\sqrt{3})^2}-\sqrt{3}\right]+\sqrt[3]{\frac{\left(\sqrt{7}^{\log_{\sqrt{7}}2}\right)^2}{\left(49^{\log_{49}\frac{1}{4}}\right)^{\frac{1}{2}}}}=$$

$$=\left[4+\sqrt{3}-\sqrt{3}\right]+\sqrt[3]{\frac{2^2}{\left(\frac{1}{4}\right)^{\frac{1}{2}}}}=4+\sqrt[3]{\frac{4}{\frac{1}{2}}}=4+\sqrt[3]{8}=4+2=\underline{\underline{6}}$$

2)

$$\frac{x^2-y^2}{2}\cdot\left(\frac{1}{(x+y)^2}-\frac{1}{(x-y)^2}\right):\left(\frac{1}{x+y}-\frac{1}{x-y}\right)=$$

$$=\frac{(x-y)(x+y)}{2}\cdot\left(\frac{(x-y)^2-(x+y)^2}{(x+y)^2\cdot(x-y)^2}\right):\left(\frac{(x-y)-(x+y)}{(x+y)\cdot(x-y)}\right)=$$

$$=\frac{\cancel{(x-y)}\cancel{(x+y)}}{2}\cdot\frac{[(x-y)-(x+y)]\cdot[(x-y)+(x+y)]}{\cancel{(x+y)}\cancel{(x+y)}\cancel{(x-y)}\cancel{(x-y)}}\cdot\frac{\cancel{(x+y)}\cdot\cancel{(x-y)}}{-2y}=$$

$$=\frac{1}{\cancel{2}}\cdot\frac{[\cancel{-2y}][\cancel{2}x]}{1}\cdot\frac{1}{\cancel{-2y}}=\underline{\underline{x}}$$

3)

$$\sqrt[3]{\frac{\sqrt{x^9\cdot\sqrt{x}}}{x^2\cdot\sqrt[8]{x}}}\cdot\sqrt[8]{x^9}=\sqrt[3]{\frac{x^{\frac{9}{2}}\cdot x^{\frac{1}{4}}}{x^2\cdot x^{\frac{1}{8}}}}\cdot x^{\frac{9}{8}}=\sqrt[3]{\frac{x^{\frac{36}{8}}\cdot x^{\frac{2}{8}}}{x^{\frac{16}{8}}\cdot x^{\frac{1}{8}}}}\cdot x^{\frac{9}{8}}=$$

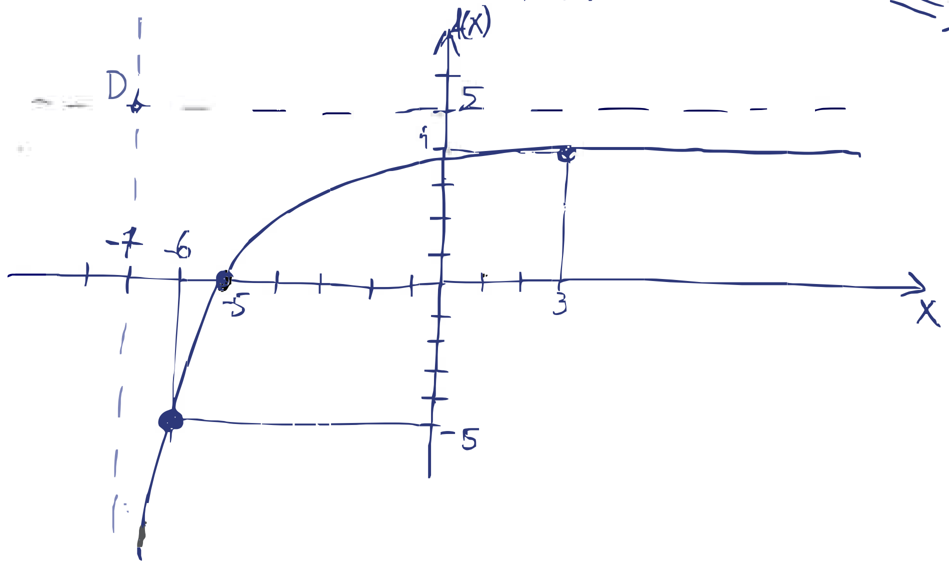
$$=\sqrt[3]{x^{\frac{36+2-16-1}{8}}}\cdot x^{\frac{9}{8}}=\sqrt[3]{x^{\frac{21}{8}}}\cdot x^{\frac{9}{8}}=x^{\frac{7}{8}}\cdot x^{\frac{9}{8}}=x^{\frac{16}{8}}=\underline{\underline{x^2}}$$

Ⓐ hétfő

4.

$$f(x)=5-\frac{10}{x+7}\quad, x>-7$$

Ábrázolható alak: $-\frac{10}{x+7}+5=f(x)$ $\qquad x>-7$



$\Rightarrow D=(-7;+5)$

Inverz:

$$y=5-\frac{10}{x+7}\Rightarrow x=5-\frac{10}{y+7}$$

$$\frac{10}{y+7}=5-x$$

$$\frac{10}{5-x}=y+7$$

$$\frac{10}{5-x}-7=y\Rightarrow \underline{\underline{f^{-1}(x)=\frac{10}{5-x}-7}}$$

5.

$$f(x)=\frac{2x^2(x-2)^2(x+1)^2-(2x^2-4x)(x^2-1)^2}{(x-2)^4(x+1)^2}$$

$x=2$ és $x=-1$ nem lehet a nevező miatt!

$\Rightarrow \boxed{D_f:\Rightarrow x\in\mathbb{R}\setminus\{-1;2\}}$

$$\frac{2x^2(x-2)^{\cancel{2}}\cancel{(x+1)^2}-[2x\cancel{(x-2)}(x-1)^2\cancel{(x+1)^2}]}{(x-2)^{\cancel{4}3}\cancel{(x+1)^2}}=\frac{2x^2(x-2)-2x(x-1)^2}{(x-2)^3}=$$

$$=\frac{2x[x(x-2)-(x-1)^2]}{(x-2)^3}\Rightarrow 2x\cdot[\cancel{x^2}-\cancel{2x}-\cancel{x^2}+\cancel{2x}-1]\overset{?}{=}0\Rightarrow \underline{\underline{x_1=0}}$$

Zérushely: $\underline{\underline{x=0}}$

B hétfő

1

$$\left(\sqrt{27+10\sqrt{2}}-\sqrt{2}\right)+\sqrt[5]{5^{\log_{\sqrt{5}}4-\log_{125}\frac{1}{8}}}=$$

$$=\left(\sqrt{5^2+2\cdot5\cdot\sqrt{2}+(\sqrt{2})^2}-\sqrt{2}\right)+\sqrt[5]{\frac{\left[(\sqrt{5})^2\right]^{\log_{\sqrt{5}}4}}{\left(125^{\frac{1}{3}}\right)^{\log_{125}\frac{1}{8}}}}=$$

$$=\left(\sqrt{(5+\sqrt{2})^2}-\sqrt{2}\right)+\sqrt[5]{\frac{\left(\sqrt{5}^{\log_{\sqrt{5}}4}\right)^2}{\left(125^{\log_{125}\frac{1}{8}}\right)^{\frac{1}{3}}}}=$$

$$=\left(5+\sqrt{2}-\sqrt{2}\right)+\sqrt[5]{\frac{4^2}{\left(\frac{1}{8}\right)^{\frac{1}{3}}}}=5+\sqrt[5]{\frac{16}{\frac{1}{2}}}=5+\sqrt[5]{32}=5+2=\underline{\underline{7}}$$

2

$$\frac{a^2-b^2}{2}\cdot\left(\frac{1}{(a-b)^2}-\frac{1}{(a+b)^2}\right):\left(\frac{1}{a-b}-\frac{1}{a+b}\right)=$$

$$=\frac{(a-b)(a+b)}{2}\cdot\left(\frac{(a+b)^2-(a-b)^2}{(a-b)^2(a+b)^2}\right):\left(\frac{a+b-(a-b)}{(a-b)(a+b)}\right)=$$

$$=\frac{\cancel{(a-b)}\cancel{(a+b)}}{2}\cdot\frac{[(a+b)-(a-b)][(a+b)+(a-b)]}{\cancel{(a-b)}\cancel{(a-b)}\cancel{(a+b)}\cancel{(a+b)}}\cdot\frac{\cancel{(a-b)}\cancel{(a+b)}}{2b}=$$

$$=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2b}\cdot2a}{1}\cdot\frac{1}{\cancel{2b}}=\underline{\underline{a}}$$

3

$$\sqrt[5]{\frac{\sqrt{x^{12}\cdot\sqrt{x}}}{x^4\cdot\sqrt[6]{x}}}\cdot\frac{1}{\sqrt[6]{x}}=\sqrt[5]{\frac{x^6\cdot x^{\frac{1}{4}}}{x^4\cdot x^{\frac{1}{6}}}}\cdot\frac{1}{x^{\frac{1}{6}}}=\sqrt[5]{\frac{x^{\frac{72}{12}}\cdot x^{\frac{3}{12}}}{x^{\frac{48}{12}}\cdot x^{\frac{2}{12}}}}\cdot x^{-\frac{1}{6}}=$$

$$=\sqrt[5]{x^{\frac{72+3-48-2}{12}}}\cdot x^{-\frac{1}{6}}=\sqrt[5]{x^{\frac{25}{12}}}\cdot x^{-\frac{1}{6}}=x^{\frac{5}{12}}\cdot x^{-\frac{2}{12}}=x^{\frac{3}{12}}=\underline{\underline{x^{\frac{1}{4}}}}$$

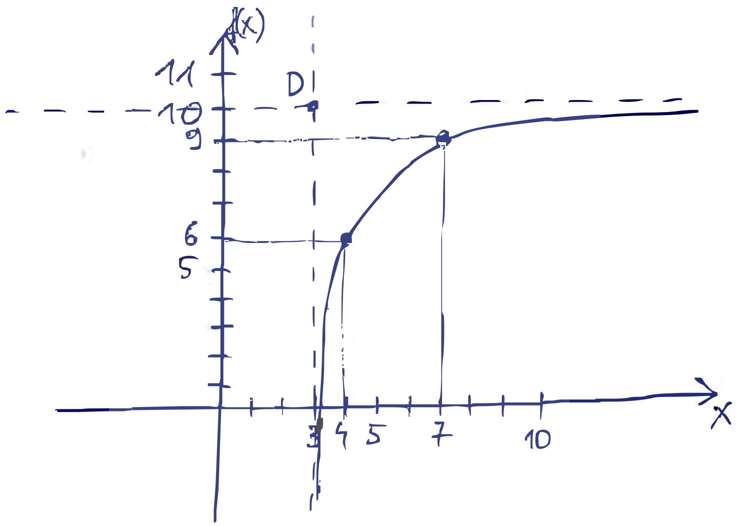
Ⓑ hétfő

④

$$f(x) = 10 - \frac{4}{x-3}, \quad x > 3$$

Ábrázolható alak: $f(x) = -\frac{4}{x-3} + 10, \quad x > 3$

$\Rightarrow D = (+3; +10)$



Inverz: $y = 10 - \frac{4}{x-3} \Rightarrow x = 10 - \frac{4}{y-3}$

$$\frac{4}{y-3} = 10 - x$$

$$\frac{4}{10-x} = y - 3$$

$$\frac{4}{10-x} + 3 = y \Rightarrow \underline{\underline{f^{-1}(x) = \frac{4}{10-x} + 3}}$$

⑤

$$f(x) = \frac{(x+2)^4(x-1)^2}{(2x^2+4x)(x^2-1)^2 - 2x^2(x+2)^2(x-1)^2}$$

Lehetséges zérushelyek a számláló miatt: $\{-2; 1\}$

$$f(x) = \frac{(x+2)^4(x-1)^2}{2x(x+2)(x-1)^2(x+1)^2 - 2x^2(x+2)^2(x-1)^2} = \frac{(x+2)^4(x-1)^2}{2x(x+2)(x-1)^2[(x+1)^2 - x(x+2)]} =$$

$$= \frac{(x+2)^4(x-1)^2}{2x(x+2)(x-1)^2[\cancel{x^2} + \cancel{2x} + 1 - \cancel{x^2} - \cancel{2x}]} = \frac{(x+2)^4(x-1)^2}{2x(x+2)(x-1)^2} \Rightarrow \left.\begin{matrix} x_1 \neq 0 \\ x_2 \neq -2 \\ x_3 \neq 1 \end{matrix}\right\} \text{A nevező miatt}$$

$$\Rightarrow \boxed{D_f: x \in \mathbb{R} \setminus \{-2; 0; 1\}}$$

$\Rightarrow$ Az $f(x)$ függvénynek nincs zérushelye!