(A)

(1)

$x^2-3|x|+x+1=0$

$\boxed{1.}$ Ha $x \geq 0$

$x^2-3x+x+1=0$

$x^2-2x+1=0$

$(x-1)^2=0$

$\underline{\underline{x_1=1}}$ ≥ 0 ✓

$\boxed{2.}$ Ha $x<0$

$x^2+3x+x+1=0$

$x^2+4x+1=0$

$x_{2;3}=\dfrac{-4\pm\sqrt{16-4}^{\sqrt{12}}}{2}$

$x_2=\dfrac{-4+2\sqrt3}{2}=\underline{\underline{-2+\sqrt3}}$ <0 ✓

$x_3=\dfrac{-4-2\sqrt3}{2}=\underline{\underline{-2-\sqrt3}}$ <0 ✓

(2)

$\left(\frac12\right)^{x^2-8x+16}<\frac18$

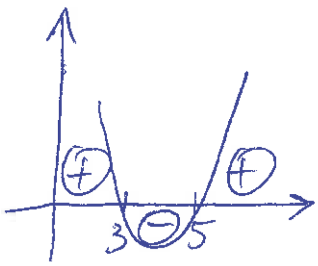
$\left(\frac12\right)^{x^2-8x+16}<\left(\frac12\right)^3$

$\Downarrow$ mivel az exponenciális függvény szig. mon. csökkenő ebben az esetben

$x^2-8x+16>3$

$x^2-8x+15>0$

$(x-5)(x-3)>0$



$\Rightarrow \boxed{x\in\,]-\infty;3[\,\cup\,]5;\infty[}$

(3)

$2\log_2(x-3)+\log_2(x^2-8x+16)=0$

$\log_2[(x-3)^2]+\log_2[(x-4)^2]=0$

$\boxed{K}$ I.) $x>3$

II.) $x^2-8x+16>0$

$(x-4)^2>0 \Rightarrow x\in\mathbb{R}\setminus\{4\}$ $x\neq4$

$\}$ $x>3$

$\log_2[(x-3)^2\cdot(x-4)^2]=\log_2 1$

$\Downarrow$ A log függvény szig. mon.

$[(x-3)(x-4)]^2=1$

$[(x-3)(x-4)]^2-1^2=0$ // $a^2-b^2=(a-b)(a+b)$ jellegű azonosság

$[(x-3)(x-4)-1][(x-3)(x-4)+1]=0$

$(x-3)(x-4)-1=0$

$x^2-7x+11=0$

$x_{1;2}=\dfrac{7\pm\sqrt{49-44}^{\sqrt5}}{2}$

$x_1=\dfrac{7+\sqrt5}{2}$ >3, $\neq 4$ ✓

$x_2=\dfrac{7-\sqrt5}{2}$ <3 ↯

$(x-3)(x-4)+1=0$

$x^2-7x+13=0$

$x_{3;4}=\dfrac{7\pm\sqrt{49-52}^{\sqrt{-3}}}{2}$

$D<0 \Rightarrow$ nincs valós megoldása

$x_1=\dfrac{7+\sqrt5}{2}$