A

1.

$$\left(\sqrt{3}-\sqrt{4+2\sqrt{3}}\right)+\left(\sqrt{3}\right)^{\log_9\left(\frac{1}{4}\right)-\log_{\frac{1}{3}}2}=$$

$$=\left(\sqrt{3}-\sqrt{3+2\sqrt{3}+1}\right)+\frac{\left(\sqrt{3}\right)^{\log_9\left(\frac{1}{4}\right)}}{\left(\sqrt{3}\right)^{\log_{\frac{1}{3}}2}}=$$

$$=\left(\sqrt{3}-\sqrt{\sqrt{3}^2+2\cdot\sqrt{3}\cdot\sqrt{1}+\sqrt{1}^2}\right)+\frac{\left(9^{\frac{1}{4}}\right)^{\log_9\left(\frac{1}{4}\right)}}{\left(\left(\frac{1}{3}\right)^{-\frac{1}{2}}\right)^{\log_{\frac{1}{3}}2}}=$$

$$=\left(\sqrt{3}-\sqrt{\left(\sqrt{3}+1\right)^2}\right)+\frac{\left(9^{\log_9\left(\frac{1}{4}\right)}\right)^{\frac{1}{4}}}{\left(\left(\frac{1}{3}\right)^{\log_{\frac{1}{3}}2}\right)^{-\frac{1}{2}}}=$$

$$=\left(\sqrt{3}-\sqrt{3}-1\right)+\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}}{2^{-\frac{1}{2}}}=$$

$$=-1+\underbrace{\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}}_{1}=\underline{\underline{0}}$$

2.

$$\sqrt[5]{\frac{x^2\cdot\sqrt[6]{x}}{\sqrt{x^3\cdot\sqrt{x}}}}\cdot\frac{1}{\sqrt{x}}=\left(\frac{x^2\cdot x^{\frac{1}{6}}}{\left(x^3\cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}}\right)^{\frac{1}{5}}\cdot x^{-\frac{1}{2}}=$$

$$=\left(\frac{x^{\frac{13}{6}}}{\left(x^{\frac{7}{2}}\right)^{\frac{1}{2}}}\right)^{\frac{1}{5}}\cdot x^{-\frac{1}{2}}=\left(\frac{x^{\frac{13}{6}}}{x^{\frac{7}{4}}}\right)^{\frac{1}{5}}\cdot x^{-\frac{1}{2}}=\left(x^{\frac{26-21}{12}}\right)^{\frac{1}{5}}\cdot x^{-\frac{1}{2}}=$$

$$=\left(x^{\frac{5}{12}}\right)^{\frac{1}{5}}\cdot x^{-\frac{1}{2}}=x^{\frac{1}{12}-\frac{1}{2}}=x^{\frac{1-6}{12}}=\underline{\underline{x^{-\frac{5}{12}}}}=\underline{\underline{\frac{1}{x^{\frac{5}{12}}}}}$$

A

$$4a^2-9b^2=(2a)^2-(3b)^2$$

3.)

$$\left(\frac{2ab}{4a^2-9b^2}+\frac{b}{3b-2a}\right):\left(1-\frac{2a-3b}{2a+3b}\right)=$$

$$=\left(\frac{2ab}{(2a-3b)(2a+3b)}+\frac{-b}{2a-3b}\right):\left(\frac{2a+3b-(2a-3b)}{2a+3b}\right)=$$

$$=\frac{2ab-b(2a+3b)}{(2a-3b)\cancel{(2a+3b)}}\cdot\frac{\cancel{2a+3b}}{2a+3b-2a+3b}=$$

$$=\frac{2ab-2ab-3b^2}{2a-3b}\cdot\frac{1}{6b}=$$

$$=-\frac{\overset{1}{\cancel{3}}b^{\cancel{2}}}{\underset{2}{\cancel{6}}\cancel{b}(2a-3b)}=-\frac{b}{2(2a-3b)}=-\underline{\underline{\frac{b}{4a-6b}}}=\underline{\underline{\frac{b}{6b-4a}}}$$

4.) Legyen Dodó eredeti testtömege: t!

Írjunk fel egyenletet a víztömegekre:

$$\underbrace{0{,}84t}_{\substack{\text{a víz tömege}\\ \text{Dodóban az}\\ \text{itatás előtt}}}+\underbrace{(800-t)}_{\substack{\text{tömegnövekedés}\\ \Rightarrow\text{elfogyasztott}\\ \text{víz tömege}}}=\underbrace{0{,}85\cdot 800}_{\substack{\text{a víz tömege Dodóban}\\ \text{az itatás után}}}$$

$$0{,}84t+800-t=0{,}85\cdot 800$$

$$800-0{,}16t=680$$

$$120=0{,}16t$$

$$t=\frac{120}{0{,}16}=\frac{12000}{16}=\underline{\underline{750}}$$

12000 : 16 = 750
80

Dodó testtömege 750 kg, amikor nagyon szomjas!

(A)

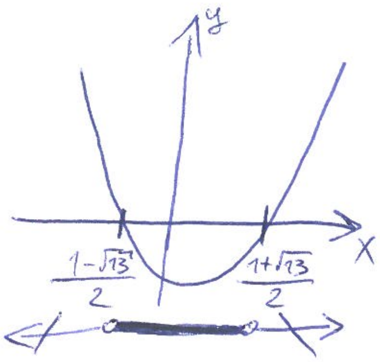
5.

$$f(x) = \ln(3 + x - x^2)$$

D_f: $3 + x - x^2 > 0$ ez kell

$$x^2 - x - 3 < 0$$

$$x_{1;2} = \frac{1 \pm \sqrt{1+12}}{2} = \begin{cases} x_1 = \dfrac{1+\sqrt{13}}{2} \\ x_2 = \dfrac{1-\sqrt{13}}{2} \end{cases} \rightarrow$$



$$\boxed{D_f: \frac{1-\sqrt{13}}{2} < x < \frac{1+\sqrt{13}}{2}}$$

Zérushely:

$$f(x) \overset{?}{=} 0 \qquad x = ?$$

$$\ln(3 + x - x^2) = 0$$

$$3 + x - x^2 = 1$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0 \Longrightarrow \underbrace{\boxed{x_1 = 2}; \boxed{x_2 = -1}}_{\text{zérushelyek}}$$

$$x_1 = 2,\ x_1 = -1 \in \left] \frac{1-\sqrt{13}}{2} ; \frac{1+\sqrt{13}}{2} \right[\checkmark$$

B

1.

$$\left(\sqrt{7}-\sqrt{8+2\sqrt{7}}\right)+\left(\sqrt{5}\right)^{\log_{\frac{1}{5}}2-\log_{25}\left(\frac{1}{4}\right)}=$$

$$=\left(\sqrt{7}-\sqrt{7+2\sqrt{7}+1}\right)+\frac{\left(\sqrt{5}\right)^{\log_{\frac{1}{5}}2}}{\left(\sqrt{5}\right)^{\log_{25}\left(\frac{1}{4}\right)}}=$$

$$=\left(\sqrt{7}-\sqrt{(\sqrt{7})^2+2\sqrt{7}\cdot\sqrt{1}+(\sqrt{1})^2}\right)+\frac{\left(\left(\frac{1}{5}\right)^{-\frac{1}{2}}\right)^{\log_{\frac{1}{5}}2}}{\left(25^{\frac{1}{4}}\right)^{\log_{25}\left(\frac{1}{4}\right)}}=$$

$$=\left(\sqrt{7}-\sqrt{(\sqrt{7}+1)^2}\right)+\frac{\left(\left(\frac{1}{5}\right)^{\log_{\frac{1}{5}}2}\right)^{-\frac{1}{2}}}{\left(25^{\log_{25}\left(\frac{1}{4}\right)}\right)^{\frac{1}{4}}}=$$

$$=\left(\sqrt{7}-\sqrt{7}-1\right)+\frac{2^{-\frac{1}{2}}}{\left(\frac{1}{4}\right)^{\frac{1}{4}}}=-1+\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}=-1+1=\underline{\underline{0}}$$

2.

$$\sqrt[3]{\frac{x^2\cdot\sqrt[10]{x}}{\sqrt{x^5\cdot\sqrt{x}}}}\cdot\frac{1}{\sqrt[5]{x}}=\sqrt[3]{\frac{x^2\cdot x^{\frac{1}{10}}}{\sqrt{x^5\cdot x^{\frac{1}{2}}}}}\cdot\frac{1}{x^{\frac{1}{5}}}=$$

$$=\sqrt[3]{\frac{x^{\frac{20}{10}}\cdot x^{\frac{1}{10}}}{\left(x^{\frac{10}{2}}\cdot x^{\frac{1}{2}}\right)^{\frac{1}{2}}}}\cdot x^{-\frac{1}{5}}=\left(\frac{x^{\frac{21}{10}}}{x^{\frac{11}{4}}}\right)^{\frac{1}{3}}\cdot x^{-\frac{1}{5}}=$$

$$=\left(x^{\frac{42-55}{20}}\right)^{\frac{1}{3}}\cdot x^{-\frac{1}{5}}=x^{-\frac{13}{60}}\cdot x^{-\frac{1}{5}}=x^{\frac{-13-12}{60}}=$$

$$=x^{-\frac{25}{60}}=\underline{\underline{x^{-\frac{5}{12}}}}=\underline{\underline{\frac{1}{x^{\frac{5}{12}}}}}$$

B

3.

$$\left(\frac{y}{2x+3y}+\frac{2xy}{9y^2-4x^2}\right):\left(\frac{2x+3y}{2x-3y}-1\right)=$$

$$=\left(\frac{y}{2x+3y}-\frac{2xy}{(2x)^2-(3y)^2}\right):\left(\frac{2x+3y-(2x-3y)}{2x-3y}\right)=$$

$$=\left(\frac{y}{2x+3y}-\frac{2xy}{(2x-3y)(2x+3y)}\right)\cdot\left(\frac{2x-3y}{2x+3y-\cancel{2x}+3y}\right)=$$

$$=\frac{\cancel{y}(2x-3y)-2x\cancel{y}}{(2x+3y)\cancel{(2x-3y)}}\cdot\frac{\cancel{2x-3y}}{6\cancel{y}}=$$

$$=\frac{2x-3y-2x}{6(2x+3y)}=-\frac{y}{2(2x+3y)}=-\frac{y}{4x+6y}$$

4.

Legyen Dodó eredeti testtömege: t!
Írjunk fel egyenletet a víztömegekre:

$$\underbrace{0{,}84t}_{\text{a víz tömege Dodóban az itatás előtt}}+\underbrace{(1200-t)}_{\text{tömegnövekedés} \Rightarrow \text{elfogyasztott víz tömege}}=\underbrace{0{,}85\cdot 1200}_{\text{a víz tömege Dodóban az itatás után}}$$

$$0{,}84t+1200-t=0{,}85\cdot 1200$$

$$1200-0{,}16t=1020$$

$$180=0{,}16t$$

$$t=\frac{180}{0{,}16}=\frac{18000}{16}=1125$$

$18000:16=1125$
20
40
80
0

Dodó testtömege 1125 kg, amikor nagyon szomjas!

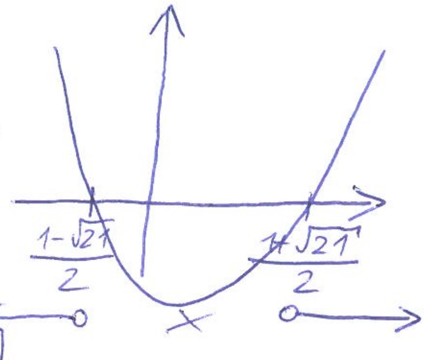
5.

B

$$f(x) = \ln(x^2 - x - 5)$$

D_f: $x^2 - x - 5 > 0$ ez kell

$$x_{1,2} = \frac{1 \pm \sqrt{1+20}}{2} \quad \begin{cases} x_1 = \dfrac{1+\sqrt{21}}{2} \\ x_2 = \dfrac{1-\sqrt{21}}{2} \end{cases} \Rightarrow$$



$$\boxed{D_f: \; x < \frac{1-\sqrt{21}}{2} \text{ vagy } x > \frac{1+\sqrt{21}}{2}}$$

Zérushely:

$f(x) = 0 \quad x = ?$

$\ln(x^2 - x - 5) = 0$ mivel a log. fv. szig. mon.

$x^2 - x - 5 = 1$

$x^2 - x - 6 = 0$

$$(x-3)(x+2) = 0 \Rightarrow \underbrace{x_1 = 3;\ x_2 = -2}_{\text{zérushelyek}}$$

$$\begin{matrix} x_1 = 3 \\ x_2 = -2 \end{matrix} \in \left]-\infty; \frac{1-\sqrt{21}}{2}\right[\vee \left]\frac{1+\sqrt{21}}{2}; +\infty\right[\checkmark$$