1 a, $i \cdot z^3 = 2i^2 - 3i^4$

$z^3 = 2i - 3i^3 = 2i + 3i = 5i$

$z^3 = 5 \cdot (\cos 90° + i \sin 90°)$ 2p

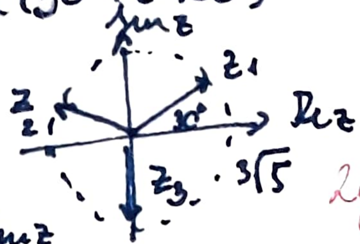
$z_{1,2,3} = \sqrt[3]{5} \cdot (\cos(30° + k \cdot 120°) + i \sin(30° + k \cdot 120°)$

$k = 0, 1, 2$

$z_1 = \sqrt[3]{5} \cdot (\cos 30° + i \cdot \sin 30°)$

$z_2 = \sqrt[3]{5} \cdot (\cos 150° + i \sin 150°)$

$z_3 = \sqrt[3]{5} \cdot (\cos 270° + i \sin 270°)$ 2p

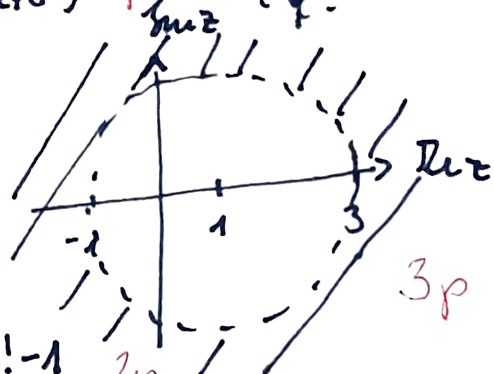


2p

b, $|z-1| > 2$ $KP(0,1)$

$|(a-1) + i \cdot b| > 2$ $R = 2$

$(a-1)^2 + b^2 > 4$ kör 3p



3p

2 ① $n = 1$ $1 \cdot 1! = 2! - 1$ ok 1p

② tfh. $n = N$-re $\sum_{k=1}^{N} k \cdot k! = (N+1)! - 1$ 2p

③ $n = N+1$ $\sum_{k=1}^{N+1} k \cdot k! = \sum_{k=1}^{N} k \cdot k! + (N+1)(N+1)! \overset{②}{=} (N+1)! - 1 + (N+1)(N+1)! =$

Ind. Felt.! $= (N+2)! - 1$ 3p

3 a, $\lim_{n \to \infty} \frac{\sqrt[3]{n^3 + 2n - 1}}{n-2} = \lim_{n \to \infty} \frac{\sqrt[3]{1 + 2/n^2 - 1/n^3}}{1 - \frac{2}{n}} = \frac{\sqrt[3]{1 + 0 - 0}}{1 - 0} = 1$ 3p 2p

b, $\lim_{n \to \infty} \sqrt[n]{3^n - 4^n + 5^n} = 5$ Rendőr-elv 1p

$5 \cdot \sqrt[n]{\frac{1}{2}} = \sqrt[n]{5^n - \frac{5^n}{2}} < \sqrt[n]{3^n - 4^n + 5^n} < \sqrt[n]{2 \cdot 5^n} = 5 \cdot \sqrt[n]{2}$

$\downarrow \quad \downarrow$ 2p $\frac{5^n}{2} > 4^n$ ha $n > 4$ $\qquad \downarrow \quad \downarrow$

$5 \cdot 1 \qquad\qquad 5 \cdot 1$ 2p

c, $\lim_{n \to \infty} \left(\frac{3n-1}{3n+2}\right)^{2n} = \lim_{n \to \infty} \left(\left(1 - \frac{3}{3n+2}\right)^{3n+2}\right)^{\frac{2n}{3n+2} \to \frac{2}{3}}$ 1p $= \left(e^{-3}\right)^{\frac{2}{3}} = e^{-2} = \frac{1}{e^2}$

$\left(1 - \frac{3}{3n+2}\right)^{3n+2} \to e^{-3}$ 3p 1p

4 $f(x) = \frac{x+2}{(x+2)(x+3)}$ $D_f = \mathbb{R} \setminus \{-2, -3\}$ 1p

$\lim_{x \to -2\pm} \frac{\cancel{x+2}}{(\cancel{x+2})(x+3)} = \frac{1}{(-2+3)} = 1$ megszüntethető szakadás (hézagpt) 2p

$\lim_{x \to -3+} \frac{1}{x+3} \to 0+ = \infty$ $\lim_{x \to -3-} \frac{1}{x+3} = -\infty$ pólus 2p

$\lim_{x \to \pm\infty} \frac{1}{x+3} = 0$ vízszintes aszimptota $y = 0$ (x-tengely) 2p